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Period–Luminosity–Colour distribution and classification of galactic oxygen–rich LPVs⋆

II. Confrontation with pulsation models

D. Barthès and X. Luri

Departament d'Astronomia i Meteorologia, Universitat de Barcelona, Avinguda Diagonal 647, 08028 Barcelona, Spain

Abstract. The kinematic and Period–Luminosity–Colour distribution of O-rich Long-Period Variable (LPV) stars of the solar neighbourhood is interpreted in terms of pulsation modes, masses and metallicities. It is first shown that, because of input physics imperfections, the periods and mean colours derived from the existing linear and nonlinear nonadiabatic models must significantly depart from the actual behaviour of the stars. As a consequence systematic corrections have to be applied, as a first approximation, to our linear model grid. These free parameters, as well as the mixing length, are calibrated on the LPVs of the LMC and of some globular clusters, assuming a mean mass of 1 $M_\odot$ for the LMC Mira-like stars. Then, the masses and metallicities corresponding to the four kinematic/photometric populations of local LPVs are evaluated. The possibility of a varying mixing-length parameter is discussed and taken into account. Stars of the old disk appear pulsating in the fundamental mode: one group, mainly composed of Miras, has mean mass $< M > \simeq 0.9\ M_\odot$ and mean metallicity $< Z > \simeq 0.02$, both strongly increasing with the period; a second group, slightly older and mainly composed of SRb's, has $< M > \simeq 0.9\ M_\odot$ and $< Z > \gtrsim 0.03$. Stars of the thin disk appear pulsating in the first and second overtones, with $< M_{\mathrm{1ov}} > \approx 1.05\ M_\odot$, $< M_{\mathrm{2ov}} > > 0.75\ M_\odot$ and $< Z > \geq 0.04$. Stars of the extended disk/halo appear pulsating in the fundamental mode, with $< M > \simeq 1.1\ M_\odot$ and $< Z > \simeq 0.01$. The mixing-length parameter probably decreases along the AGB by no more than 15% per magnitude. The large, positive period corrections (more than 30% for the fundamental and 8% for the first overtone) that have to be applied to the LNA models used in this study do not seem to be explained by imperfect sub-photospheric physics alone, especially when nonlinear effects are taken into account. The origin of the extra period increase (at least 15% for the fundamental mode) may be the stellar wind, which was neglected by all pulsation codes up to now.

Key words. stars: variables: Long Period Variables – AGB – fundamental parameters – oscillations

1. Introduction

Among the most important and still unsolved issues concerning Long-Period Variable stars (LPV) are the modelling of their pulsation and even the mere identity of the predominant pulsation mode. Linear nonadiabatic (LNA) models are currently used on a wide scale for reasons of simplicity. Relying on LNA relations between the period and the fundamental parameters, and on dynamical models of the photospheric region, the pulsation mode of Mira stars is the fundamental according to the observed velocity amplitude (Hill & Willson 1979; Willson 1982; Bowen 1988; Wood 1990), and the first overtone according to the angular diameter estimates (Haniff et al.

1995; Van Leeuwen et al. 1997), pulsation accelerations (Tuchman 1991) and the PL distribution of globular cluster stars (Feast 1996). One study based on the periods and luminosities of LPVs of the Large Magellanic Cloud (LMC) concluded that Miras were pulsating in the fundamental mode and low-amplitude semi-regulars on the first or second overtone (Wood & Sebo 1996). However, another study, also taking into account a few Miras of the solar neighbourhood and various temperature estimates, supported the first overtone in a majority of Miras (Barthès 1998). Last, still relying on LNA models, Barthès & Tuchman (1994) and Barthès & Mattei (1997) found that the Fourier spectra of a few Miras were more easily explained than were the stars pulsating on the first overtone with the fundamental mode and other overtones also acting.

Send offprint requests to: D. Barthès
⋆ Based on data from the HIPPARCOS astrometry satellite.

In fact, the large pulsation amplitude and the complexity of the chemistry and radiative transfer in the photospheric region make the mean effective temperature and radius very uncertain, even for nearby stars (Bessell et al. 1989a, 1996; Hofmann et al. 1998; Ya'ari & Tuchman 1998). The HIPPARCOS parallaxes, too, usually have large error bars, and they concern only about two dozen Miras.

Last, most of these studies suffered from a strong sampling bias, favouring periods longer than 250 days, especially those ranging from 300 to 400 days.

On the other hand, hydrodynamical models predict that, after thermal relaxation, Miras pulsate in the fundamental mode with a period either much shorter than (Ya'ari & Tuchman 1996, 1999) or very close to (Wood 1995; Bessell et al. 1996; Hofmann et al. 1998) the LNA period. These models mainly differ by their handling of the convective energy transport and of the equation of state, but they share the same unrealistic assumption: no wind and no extended circumstellar envelope at the outer boundary. As a consequence, the reliability of both the nonlinear and linear nonadiabatic pulsation models is uncertain, and the pulsation mode is still ambiguous.

An unfortunate result of these theoretical difficulties is that the present masses and metallicities of the LPVs have always been very uncertain. Indeed, no direct estimate of mass in binary systems has been possible up to now, and the chemistry and radiative transfer in the photospheric region are so complex that no reliable metallicity has been derived.

This paper mainly consists in exploiting the classification (based on kinematic and photometric criteria) and the period–luminosity–colour (PLC) distributions of the oxygen-rich LPVs of the solar neighbourhood, which were determined in Paper I of this series (Barthès et al. 1999). Its aims are to assess the pulsation models, and to identify the predominant mode and estimate the average mass and metallicity for each kinematic/photometric group.

The data sets are presented in Sect. 2. They include the results of Paper I, i.e. the periods, absorption-corrected absolute magnitudes and de-reddened colours of a sample of stars observed by HIPPARCOS, but also the de-biased distributions of the four kinematic/photometric groups to which they belong. LPVs found in the LMC and in various globular clusters are also included, with a view to calibrating the free parameters of the models.

Section 3 describes the pulsation models to which these data will be confronted: the linear nonadiabatic modelling code and the adopted temperature scale, i.e. colour–temperature–metallicity (CTZ) relations. Section 4 then explains the difficulties to be expected because of the physical approximations of the models, and also because of the nonlinear shape of the CTZ relations. We will be led to introduce some systematic correction parameters for the periods and colours.

These free parameters, together with the mixing length, are calibrated in Sect. 5 by confronting the models with the PLC distributions of the LPVs in the LMC and globular clusters. The so-calibrated models are then confronted with the PLC distribution of local stars in Sect. 6, and the pulsation mode, mass and metallicity are derived for each kinematic/photometric group. Finally, the stability and reliability of the results, as well as their consequences concerning the existing pulsation codes, are assessed in Sect. 7.

2. Data

2.1. Solar neighbourhood

Paper I has provided us with the mean absolute K_0 magnitudes and $(V - K)_0$ indices of about 350 M-type LPVs of the solar neighbourhood. For about 250 stars, $(J - K)_0$ is available. The K magnitude is known to closely mimic the behaviour of bolometric magnitude. Moreover, as mentioned in Paper I, simple simulations of light curves have shown that the mean K magnitude and the magnitude corresponding to the mean K flux agree with one another to within a few percent (0.06 mag for a full amplitude of 1 mag). The adopted magnitude is thus a good representative of the actual mean luminosity of each star. We must recall however, that, because of the paucity of the data, which were taken at arbitrary phases, the observational error on K is $\sigma = 0.25 - 0.50$ mag.

The mean $V - K$ index was defined in Paper I as the difference between the midpoint V magnitude and the mean K magnitude. Simulations have shown that the midpoint value of V is systematically larger than the mean by, usually, only a few hundredths of mag, or at the very most a few tenths. On the other hand, the magnitudes at minimum brightness tend to be underestimated by visual observers. However, this systematic error (a few tenths of mag; the half for the mean) depends altogether on the distance, the mean absolute magnitude and the amplitude (which is correlated to the brightness, but with an important scatter). Thus, globally over the sample of stars and the subsamples defined in Paper I, the systematic error on $< V >$ should not exceed -0.1 mag and thus may be neglected. Besides, the random error is about 0.3 mag. Summarizing, the $V - K$ data used in Paper I are subject to a random error of about 0.5 mag.

The luminosity calibrations performed in that paper led us to identify four groups, differing by their kinematics and/or luminosity:

Group 1: mainly composed of Mira stars (84%), with a kinematics corresponding to old disk stars;

Group 2: mainly composed of SRb stars (56%), also with old disk kinematics;

Group 3: mainly composed by SRb stars (82%), with a younger kinematics;

Group 4: a small group (14 stars, including 13 Miras) with a kinematics corresponding to extended-disk or halo stars.

The calibration procedure also provided us with the de-biased distributions of the populations corresponding to these groups. As explained in Paper I, the overall error

bars of their parameters were computed by means of many Monte-Carlo simulations, taking into account the above-mentioned photometric errors, the astrometric ones and those of the radial velocities. Concerning the barycenters of these distributions (i.e. the population means), the uncertainties are: about 0.1 mag (Groups 1, 2 and 3) or 0.25 (Group 4) for $(V-K)_0$; about 0.05 (Groups 1 and 2), 0.08 (Group 3) or 0.1 (Group 4) for M_K.

Last, it must also be mentioned that, for a minor but significant proportion of semi-regular stars, the periods may by erroneous by as much as a factor of 2, either because there are few data and/or we could not check them (see Paper I), or because the light curves and/or Fourier spectra are ambiguous (two or three large-amplitude pseudo-periodicities liable to correspond to a mode – see Mattei et al. 1997). This means that the period scattering in the groups including a large proportion of semi-regulars (i.e. Groups 2 and 3) is probably overestimated. Here again, the population mean should remain nearly unaffected.

2.2. Globular cluster stars

For oxygen-rich LPV stars belonging to globular clusters (GC) of our Galaxy, periods, mean absolute bolometric magnitudes (derived from blackbody fits to mean, dereddened $JHKL$ data) and mean $(J-K)_0$ index values have been found in Whitelock (1986) and Feast (1996). The $< m_{bol} >$ precision is about 0.1–0.3 mag according to the amplitude and data sampling. That for $J-K$ is 0.03–0.15 as explained above, but most often $\simeq 0.05$.

2.3. LMC stars

As hundreds of O-rich Mira-like stars (i.e. LPVs with $\Delta I \leq 0.9$ mag) have been observed in the Large Magellanic Cloud, we will handle them in a synthetic way, by considering their mean Period-Luminosity (M_{bol}) and Period-Colour ($J-K$) relations, the 1σ scattering about it, and the barycenter of this population. The latter is defined by the mean period, already computed by Reid et al. (1995) and the corresponding mean bolometric magnitude and colour. The PL and PC relations were taken from Feast et al. (1989) and Hughes & Wood (1990) and hold as long as $P \lesssim 420$ days. Due to the large number of stars, the error bars of the barycenter may be neglected.

We have also derived from Wood & Sebo (1996) mean K_0 magnitudes and $(J-K)_0$ colours of O-rich LPVs found near a few clusters of the LMC (2 data points per star, taken at arbitrary phases). The magnitudes were then converted into bolometric ones by applying the empirical bolometric correction $BC_K = f((J-K)_0)$ given by Bessell & Wood (1984). The obtained precision, including intrinsic variability effects, should be roughly 0.3 mag for $< K >$ and ≤ 0.10 for $< J-K >$. Two thirds of these stars are obviously pulsating on a higher-order mode than the Mira-like population, since they form a second,

parallel strip in the PL plane. That is why we included this sample among the LMC data.

Last, K magnitudes and periods of hundreds of pseudoperiodic red variables belonging to the MACHO sample have been found in Wood (1999). Being single-phase observations, these data represent the mean magnitude within about 0.3–0.5 mag. On the other hand, the periods of these stars are secure, since they were derived from MACHO light curves spanning years.

3. Modelling

3.1. LNA pulsation models

The AGB linear nonadiabatic (LNA) pulsation models used in this study are based on the code of Tuchman et al. (1978), modified as explained in Barthès & Tuchman (1994). The equation of state (EOS) includes the radiation and an ideal gas of e^-, H_2, H_2^+, H, H^-, H^+, He, He^+ and He^{++}, as well as a few heavy elements, while abundances are determined by solving the Saha equation. Convection is treated according to the mixing-length formalism of Cox & Giuli (1968), with instantaneous adjustment to pulsation. Recent opacity tables, assuming solar composition and including molecules at low temperature, namely OPAL92 and Alexander (1992) (see Alexander & Ferguson 1994), are used.

The grid of models assumes $X = 0.7$ and covers metallicities $Z = 0.02$ and 0.001, masses ranging from 0.8 to 2 $M_\odot$, luminosities ranging from 1000 to possibly 50 000 $L_\odot$, and three values of the mixing–length parameter: $\alpha = 1$, 1.5 and 2. Then, a log-linear least-squares fit (excluding the extreme luminosities where iso-mass lines would turn back) gives us theoretical relations between the effective temperature or pulsation mode periods on the one hand, and the mass, metallicity, luminosity and mixing-length parameter on the other (MLZαT and MLZαP relations). Between $\alpha = 2$ and 3, the extrapolated periods and effective temperatures are precise within 1–2%.

3.2. Temperature scale and bolometric correction

Comparison between the theoretical models and observational data requires one to convert effective temperatures into $V-K$ or $J-K$ indices.

Bessell et al. (1989a, 1996) have computed dynamical models of Mira atmospheres with various fundamental parameters (but always *solar* metallicity) and derived many colour indices, including $V-K$ and $J-K$, at various phases, thus various effective temperatures (up to 3770 K), gravities and atmospheric extensions. Recently, Bessell et al. (1998) have published broad-band colours derived from *static* models of giant star atmospheres for temperatures higher than 3600 K. Comparison with the older static models of Bessell et al. (1989b) (which were the basis of their dynamical models) shows that the recent ones are systematically redder by a few 10^{-1} mag
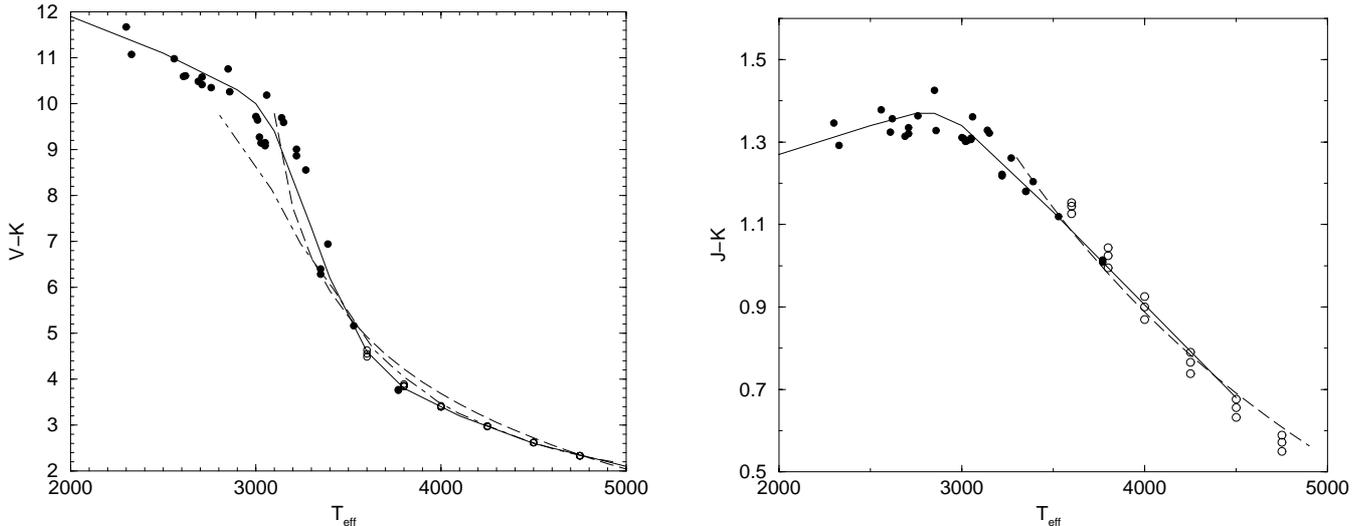
Fig. 1. a) Adopted relation between $V - K$ and effective temperature (full line), compared to dynamical models of Bessell et al. (1989a, 1996) (filled circles), static models of Bessell et al. (1998) (open circles) and the empirical non-LPV relations of van Belle et al. (1999) (dashed line) and Perrin et al. (1998) (dot-dashed line). Solar metallicity assumed. **b)** The same for $J - K$. The empirical non-LPV relation (dashed line) is from Bessell et al. (1983)

for $V - K$ and a few 10^{-2} mag for $J - K$ in the small overlapping domain of temperature and gravity. This is due to differences in the opacities and their handling.

On the other hand, van Belle et al. (1999) have published an empirical relation between $V - K$ and T_{eff} (above 3030 K) derived from the interferometric angular diameters of 59 *non-LPV* giant stars (non-variable or with a very small V amplitude). Similar work, extending to lower temperatures, was also performed by Perrin et al. (1998). Moreover, an empirical relation involving $J - K$ has been published by Bessell et al. (1983), again for non-LPV stars.

Considering the inconsistency of all these sources, there is no better way to derive a colour–temperature (CT) relation for each colour index than to perform a simple eyeball fit to the model data, as shown in Fig. 1. This takes into account the above-mentioned dynamical models of Bessell et al. (1989a, 1996), and the static models of Bessell et al. (1998) at gravities $\log g = -0.5$, 0 and $+0.5$. Roughly, one may estimate that our empirical relations are precise within 0.5 mag for $V - K$ and 0.05 for $J - K$ (subject to possible systematic error due to the imperfect modelling of the molecular lines).

Of course, it was necessary to correct the CT relations for intrinsic metallicity-dependence. The only information that we could find in the literature originated from static models. So, at temperatures ≤ 3350 K, we applied a parabolic $\{\Delta(\log Z), \Delta(\text{colour})\}$ fit and a series of linear $\{T_{eff}, \Delta(\text{colour})\}$ interpolations to the models XX, X, Y and YY of Bessell et al. (1989b). They correspond respectively to $[M/H] = +0.5$, 0, -0.5 and -1, and to $\log g$ evolving from -1.02 to -0.43 as the temperature increases, so as to mimic the AGB. At $T_{eff} \geq 3600$ K, we proceeded in the same way with the models listed in Table 5 of Bessell et al. (1998), while extrapolating the

gravity sequence initiated by the models of Bessell et al. (1989b).

Around 3480 K, the $\{T_{eff}, Z, V - K\}$ relation resulting from Bessell et al. (1989b) exhibits a crossing-over that appears only around 3800 K in the Bessell et al. (1998) models. As a consequence, the $V - K$ correction was simply interpolated between 3350 and 3600 K, without considering the intermediate model data. This does not concern $J - K$.

On the other hand, wherever necessary (see Sect. 6), we have converted the theoretical bolometric magnitudes into K magnitudes, by subtracting the the empirical bolometric correction $BC_K = f(V - K)$ given by Bessell & Wood (1984). The used $V - K$ is, of course, the value derived from the model temperature.

4. Is theory directly comparable to observations?

Comparing theoretical models with observational data, with a view to calibrating the internal parameters of the theory and to deriving physical information on the stars (e.g. the pulsation mode and the fundamental parameters) does not make sense if the models are basically wrong or if their predictions are systematically shifted because of imperfections of their input physics. We thus have to assess the reliability of the model grid, including the temperature scale, and to find a way to (partially) compensate its defects.

4.1. Static/LNA modelling

First, one may remember that the core mass–luminosity relation (CMLR) assumed in our calculations corresponds to maximum hydrogen shell luminosity (Paczyński 1970). In fact, even neglecting the very short helium flashes, the

luminosity varies by a factor of two over a thermal pulse cycle, and the CMLR approximately holds over only about 25% of the interflash time (Boothroyd & Sackman 1988a, 1988b; Wagenhuber & Tuchman 1996). For a global investigation of our sample of stars, we would perhaps do better adopt an "effective" CMLR that would be, say, 30% less luminous. Doing so, at given total mass and luminosity, the effective temperature becomes 1% higher, and the period decreases by a few %.

Our calculations also assume that the convective flux and velocity instantly adjust themselves to pulsation, i.e. that the mean eddy lifetime is about zero. In fact, it is roughly a third of the fundamental period (Ostlie & Cox 1986), which induces a significant phase lag. In order to estimate the resulting uncertainty, we have recomputed some of our models while assuming a frozen-in convection, i.e. infinite eddy lifetime. The obtained fundamental periods are shorter by 5–10%, and the first overtone longer by 1–4%. In other terms, as long as convection phase lag is concerned, the fundamental periods predicted by our model grid are probably overestimated by roughly 5%, while the first overtone is underestimated by perhaps 2%.

On the other hand, Ostlie & Cox (1986) have attempted to improve the standard LNA modelling by a horizontal opacity averaging scheme, which accounts in a simplified way for the coexistence of rising and falling convective elements in the same mass shell. Periods then increase by less than 10%. Moreover, the same authors have investigated the effects of turbulent pressure: the obtained period shifts range from +8 to +36% for the fundamental mode and from +3 to +11% for the first overtone. Turbulent viscosity and energy have negligible effects (Cox & Ostlie 1993).

Summarizing, one may expect the model grid described in Sect. 3 to underestimate the fundamental period by as much as 40% for the fundamental mode and 25% for the first overtone, and the relative shift of the latter mode is always more than a third of the former.

It is worth noting that the often quoted models of Wood (Wood 1974; Fox & Wood 1982; Wood 1990; Bessell et al. 1996; Hofmann et al. 1998; Wood et al. 1999), which include a phase-lagged convection scheme, unfortunately use an outdated equation of state. As far as we know, this is the only important difference with Tuchman's code. The EOS is thus probably the reason why Wood's linear fundamental and first overtone periods are longer by 15–70% than the ones predicted by our models, with the same opacity tables and composition regardless of our treatment of convection[1]. The relative shift is always about the same for the two modes. Interestingly, the fundamental period shift is the same order of magnitude as that wich would result from opacity averaging and turbulent pressure together. As a consequence, Wood's fundamental pulsation models can often reach a reasonable agreement with the observations, but first overtone

periods derived from this code are strongly overestimated, as also noted by Xiong et al. (1998).

4.2. Nonlinear effects

The periods of the pulsation modes predicted by LNA models correspond to small-amplitude oscillations of the static stellar envelope. The effective temperature, too, is that of the static star. In fact, the large-amplitude pulsation of an LPV is strongly nonlinear and makes the outer envelope expand. As a consequence, the mean value of the effective temperature, as well as the periods and growthrates of the pulsation modes, do not necessarily equal the values given by a linear model of the same star.

According to the various hydrodynamical calculations performed up to now (Wood 1974; Tuchman et al. 1979; Perl & Tuchman 1990; Tuchman 1991; Ya'ari & Tuchman 1996, 1999; Bessell et al. 1996) the static effective temperature may differ from the mean T_{eff} of the corresponding pulsating star by plus or minus 1–5%.

On the other hand, some recent calculations (Ya'ari & Tuchman 1996, 1999) have shown that the period of the nonlinear fundamental mode may, after thermal relaxation, be *shorter* than the LNA value by as much as 35% (depending, at least, on the luminosity). But models based on Wood's code (Wood 1995; Bessell et al. 1996; Hofmann et al. 1998) predict only a small *increase* of the fundamental period, even after spontaneous thermal relaxation (models P, M and O of Hofmann et al. 1998). As stated above, the main difference between these two families of models are the treatment of time-dependent convection and the equation of state, which both have important thermal effects. As phase-lagged convection tends to increase the nonadiabaticity of the pulsation, it is likely that the EOS is the main cause of the more "quiet" behaviour of Wood's models (as long as the numerical scheme is not at stake).

4.3. Outer boundary

Ya'ari & Tuchman (1996) report that their nonlinear results appear basically insensitive to the various outer boundary conditions that they have tried. However, all abovementioned models neglect the fact that dust condensates in the levitating circumstellar layers and, as an effect of radiation pressure, generates a significant stellar wind, ranging from 10^{-8} to 10^{-6} $M_{\odot}$/yr for Miras and semi-regulars (Jura 1986; Jura 1988; Jura et al. 1993; Kerschbaum & Hron 1992). The physics of this phenomenon was extensively described by Fleischer et al. (1992) and Höfner & Dorfi (1997).

Due to the extension of the envelope, the outgoing waves are only partially reflected in the photospheric region, and this does not occur at a fixed level but at the sonic point, which depends on the wind. Pijpers (1993) has shown, in the case of polytropic AGB star models, that the adiabatic fundamental period may increase by

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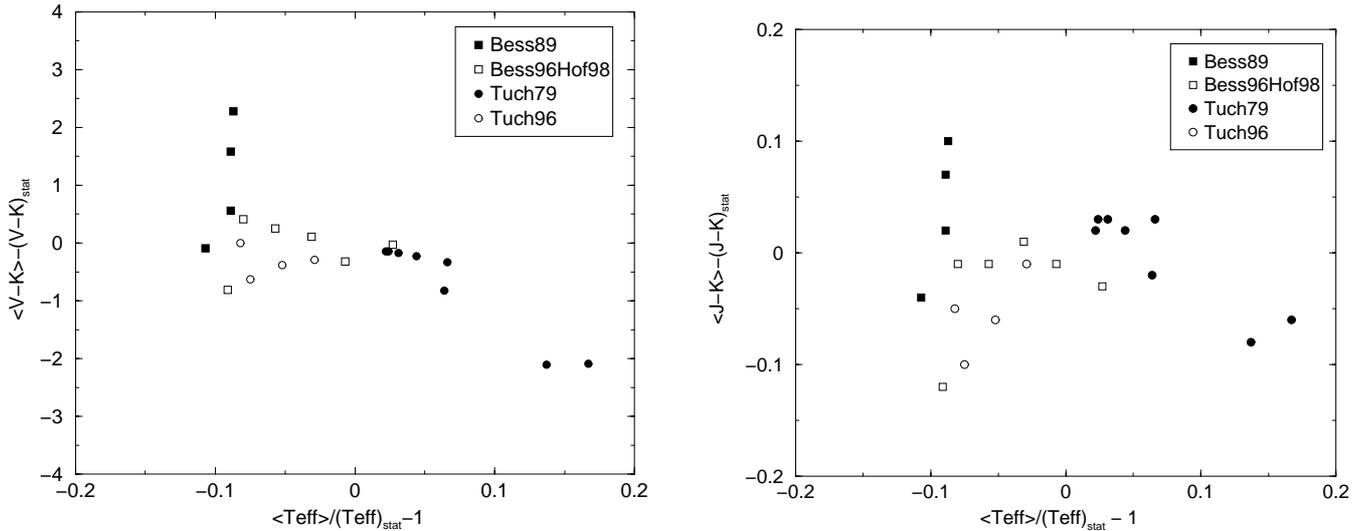[1] Wood's periods increase much faster than ours with luminosity and metallicity.

Fig. 2. Difference between the mean colour index and the colour at the static temperature, according to various dynamical models (see text). Left: $V - K$; right: $J - K$

more than a factor 5 if a mass-loss rate of 10^{-6} $M_\odot$/yr is assumed. It would be surprising if such a large shift of the adiabatic periods had no effect on the nonadiabatic ones. Even though polytropic models are just a rough approximation, one may also expect some effects of this kind in real stars.

4.4. Temperature scale

The nonlinear shape of the temperature–colour relations is another source of difficulty: it may generate a significant mismatch between the mean colour index and the colour corresponding to the mean temperature, depending on the latter and on the pulsation amplitude.

It is thus clear that a linear model having the fundamental parameters of a given LPV (in particular its static temperature), the correct mixing length, and predicting the correct period, will nevertheless disagree with the observed colours by a significant amount. In order to estimate the overall colour mismatch ($< V - K > -(V - K)_{\text{stat}}$, $< J - K > -(J - K)_{\text{stat}}$) that may be expected we have applied the above-defined temperature–colour relations to simulated temperature oscillations based on the dynamical models of Tuchman et al. (1979), Bessell et al. (1989), Bessell et al. (1996), Ya'ari & Tuchman (1996) and Hofman et al. (1998), which have very diverse fundamental parameters and amplitudes (usually 1 $M_\odot$, but up to 6 $M_\odot$ in Tuchman et al. (1979); 1800–35000 $L_\odot$; 2250–3500 K (static); 15–40 km s^{-1}). The results are shown in Fig. 2. The mean shifts and standard deviations are -0.15 and 0.95 for $V - K$ (but 4/5 of the models lie within 0.7 of the mean), and -0.01 and 0.05 for $J - K$. No obvious correlation with any fundamental parameter or with the pulsation period or amplitude was found. It is likely that the colour mismatch depends on a non-trivial combination of these parameters.

4.5. Consequences

All this discussion leads us to conclude that no linear model grid can be expected to directly fit the observations, and that the existing nonlinear models are not yet able to provide a reliable grid, or even an a priori correction of the linear models. As a consequence, linear models must be complemented by additional free parameters, to be added prior to comparison with the observations. As a first-order approximation, these parameters can be some systematic corrections of the colour ($\Delta(V - K)$ or $\Delta(J - K)$) and, for each pulsation mode, of the period ($\Delta \log P$). Before trying to interpret the results of Paper I, these parameters and the mixing length have to be calibrated by fitting the models to independent data, namely the LPVs observed in globular clusters and in the LMC.

5. Models calibration: Clusters and LMC stars

The sample of globular cluster stars can be divided in three sets: one with the metallicity of the LMC ($Z = 0.008$); a second one with about the metallicity of the SMC ($Z = 0.004 \pm 0.001$); and the last one with $Z \approx 0.0005$. Only the two first sets (hereafter called LMC-like and SMC-like) will be used, because the adopted log-linear model fit and colour–temperature relation are no longer reliable at very low metallicities and masses. In fact, each set may be divided in two sub-sets, obviously corresponding to two different pulsation modes (they are well separated in the PL and PC planes, see Fig. 5). Each set will be represented by two points, namely the barycenters (mean periods, magnitudes and colours) of its two sub-sets[2]. This facilitates the model fitting, reduces the observational error bars down to low levels (≤ 0.03 on

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[2] The calculation of the means excludes one SMC-like star lying right in between the two subsets, and three outlying LMC-like stars.
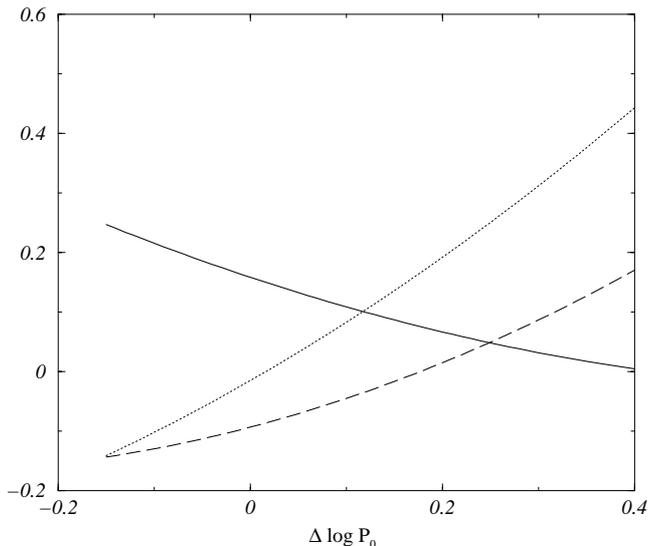
Fig. 3. The $\Delta(J - K)$ colour correction parameter (dashed line), the mixing-length parameter $\alpha/2 - 1$ (dotted line) and the mass discrepancy of the fundamental mode in SMC-like clusters δM (solid line) as a function of the period correction parameter $\Delta \log P_0$

$J - K$), and reduces the possible effects of the scattering of $< J - K > - (J - K)_{\text{stat}}$ down to $\sigma \lesssim 0.015$. In fact, there are only four stars in LMC-like globular clusters. We thus prefered to merge them with the sets of LMC stars (i.e. two with the Mira-like stars and two with the higher-order pulsators).

As an additional constraint, we assume that the mean mass of the Mira-like population of the LMC is $1 M_\odot$. This ensures that, whatever the choice of the free parameters, the masses of the sample LPVs of the LMC do not exceed $1.5 M_\odot$, consistent with the evolutionary calculations (Wood & Sebo 1996). Then, for every value of $\Delta \log P$, the theoretical MLZαT and MLZαP relations give us the single possible value of α and of $\Delta(J - K)$. The pulsation mode that gives the better fit to Mira-like stars is always the fundamental.

Then, keeping the three parameters unchanged, we obtain two masses for the barycenter of the SMC-like subset corresponding to the fundamental mode: one derived from the period, the other from the colour. The calibration thus consists in minimizing the difference δM of these two masses. As shown in Fig. 3, the mass discrepancy decreases as the period (and colour) shift increases. The model fit starts being acceptable ($\delta M \leq 0.1\ M_\odot$) at $\Delta \log P_0 = +0.13$. The mean colour shift derived from the dynamical models (Fig. 2), viz. $\Delta(J - K) = -0.01$ and thus $\Delta \log P_0 = +0.16$ and $\alpha = 2.29$, gives a reasonable fit: $\delta M = 0.07\ M_\odot$. We adopt this solution, for which the most probable value of $\Delta(V - K)$ is already known (-0.15, of course, i.e. the mean of the a priori estimates of Sect. 4). Indeed, considering all uncertainties (especially concerning $< J - K >$ and the CTZ relation) as well as the lack of solid theoretical ground, it would make little

sense to look for an exact agreement of the masses by further increasing the correction parameters (see Sect. 7 for further discussion).

As a last step, we can now determine the period correction of the first overtone, which must ensure that consistent masses are obtained from the MLZαT and MLZαP relations at the barycenters of the subsets corresponding to this mode. We obtain $\Delta \log P_1 = 0.056$, with negligible mass discrepancies.

The so-calibrated model grid is represented in Figs. 4 and 5 by a series of isomass lines (0.6, 0.8, 1 and $1.5\ M_\odot$). The results of this calibration are summarized in the upper-left quarter of Table 1. Let us however recall that masses as low as $0.6\ M_\odot$ are probably slightly underestimated by the log-linear fit scheme.

The likelihood of the calibrated model grid can be checked by confronting it to the K magnitudes and periods of the MACHO sample of Wood (1999). As can be seen in Fig. 6, the strip corresponding to Mira-like stars still fits the fundamental mode, with mean mass $1\ M_\odot$. The strip immediately on its left fits the first overtone. Last, a third strip obviously corresponds to the second overtone. We could not calibrate its correction parameter but, having noticed that $\Delta \log P_1 \simeq \frac{1}{3}\Delta \log P_0$, we adopted $\Delta \log P_2 = \frac{1}{3}\Delta \log P_1$. Our interpretation is consistent with that of Wood et al. (1999) and Wood (1999), who also considered the stars lying on the right of the figure as probable binaries or stars pulsating on a thermal mode coupled to the fundamental.

6. Application to the solar neighbourhood

Having fully calibrated the models, we can now investigate the results of Paper I, i.e. the four de-biased PLC distributions of the local LPVs and the calibrated and de-biased individual data (sample stars). For the sake of clarity, members of the old disk (Groups 1 and 2 as defined in Paper I), thin-disk (Group 3) and exended-disk/halo (Group 4) populations will be separately investigated. Mathematically speaking, the work to be done consists in determining the pulsation mode, mass and metallicity corresponding to the barycenter of the de-biased distribution of each group, by solving the MLZαT and MLZαP equations with the above-calibrated free parameters.

At each barycenter, the absolute K magnitude was converted into the bolometric one by applying the bolometric correction defined in Sect. 3.2, using the corresponding $< (V - K)_0 >$. Then, the period and colour correction parameters ($\Delta \log P = 0.16$ or 0.056 and $\Delta(V - K) = -0.15$) were subtracted from the data points before solving the two equations. It must be mentioned, however, that Figs. 7 through 11 were plotted keeping the observations unchanged, i.e. using K magnitudes. We thus applied to the models bolometric corrections that were deduced from the theoretical $V - K$ after adding $\Delta(V - K)$. The results are detailed in the next subsections and summarized in the left-hand part of Table 1. The isomass lines in the figures correspond to 0.8, 1 and $1.5\ M_\odot$.

Table 1. Mean theoretical masses and metallicities of the fundamental, first overtone and second overtone pulsators of the LMC, Globular Clusters with SMC metallicity and the solar neighbourhood. The mixing-length parameter is assumed to be constant (left) or to decrease by 35% every -2 bolometric magnitudes along the AGB (right)

|	$\alpha = 2.29$			$\alpha = f(L)$		
	Mode	$< Z >$	$< M >$ ($M_\odot$)	Mode	$< Z >$	$< M >$ ($M_\odot$)
LMC (Mira-like)	F	0.008	1.00	F	idem	idem
LMC	1ov	0.008	0.95	1ov	idem	0.95
Z_{SMC} GC	F	0.004	0.8	F	idem	0.75
Z_{SMC} GC	1ov	0.004	0.6	1ov	idem	0.5
Group 1 pop.	F	0.020	0.9	F	0.020	0.9
sample	F	0.04	1.7	F	0.024	1.85
Group 2 pop.	F	0.027	0.95	F	0.07	0.85
sample	F	0.035	1.2	F	0.05	1.1
Group 3 pop.	1ov	0.04	1.1	1ov	0.07	1.0
	2ov	0.04	0.75[a]	—	—	—
Group 4 pop.	F	0.009	1.1	F	0.010	1.1
sample	F	0.014	1.65	F	0.010	1.7
$\Delta \log P_0$	+0.16			+0.126		
$\Delta \log P_1$	+0.056			+0.033		
$\Delta \log P_2$	+0.019			+0.009		
$\Delta(J - K)$	-0.01			-0.065		
$\Delta(V - K)$	-0.15			-0.53		

[a] Lower boundary.
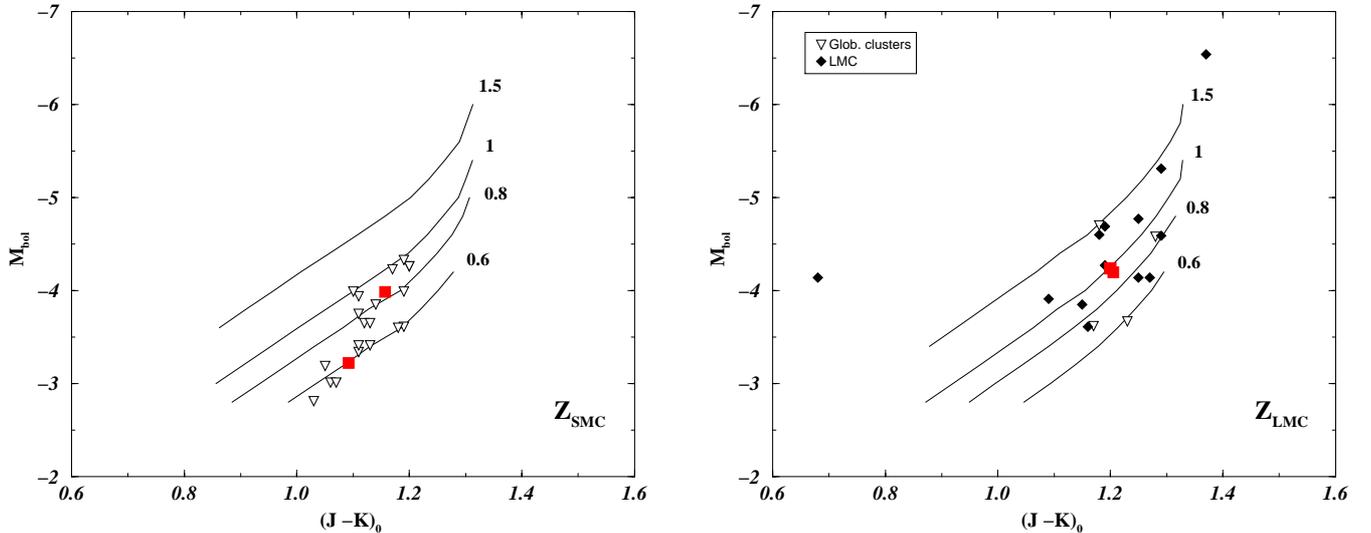
Fig. 4. Left: the LC distribution of LPVs in globular clusters with SMC-like metallicity compared to the calibrated models, assuming $Z = 0.004$. The two barycenters are indicated by filled squares. Right: the same for clusters with LMC-like metallicity and for the barycenter of the Mira-like population of the LMC with $P \leq 420$ d; $Z = 0.008$ assumed

The de-biased 3D distribution of each Group, shown in Fig. 7, appears as a quasi-elipsoidal volume containing 60% of the population. In the three fundamental planes, it is represented by a quasi-elliptical contour which is the projection of the elliptical 2σ iso-probability contour defined in the main symmetry plane (see Paper I). Then, the pulsation mode and the mean mass and metallicity are given by the barycenter of the de-biased distribution, i.e. the center of the "ellipsoid" or "ellipse": if the models have been well calibrated and if their adopted metallicity equals the actual population mean, the surface corresponding to one theoretical mode in the 3D diagram must include the barycenter. In the three 2D figures, this point must lie on the same iso-mass line, which property was used above for calibrating the models. Another advantage of working on the barycenter is that we avoid the projection effects
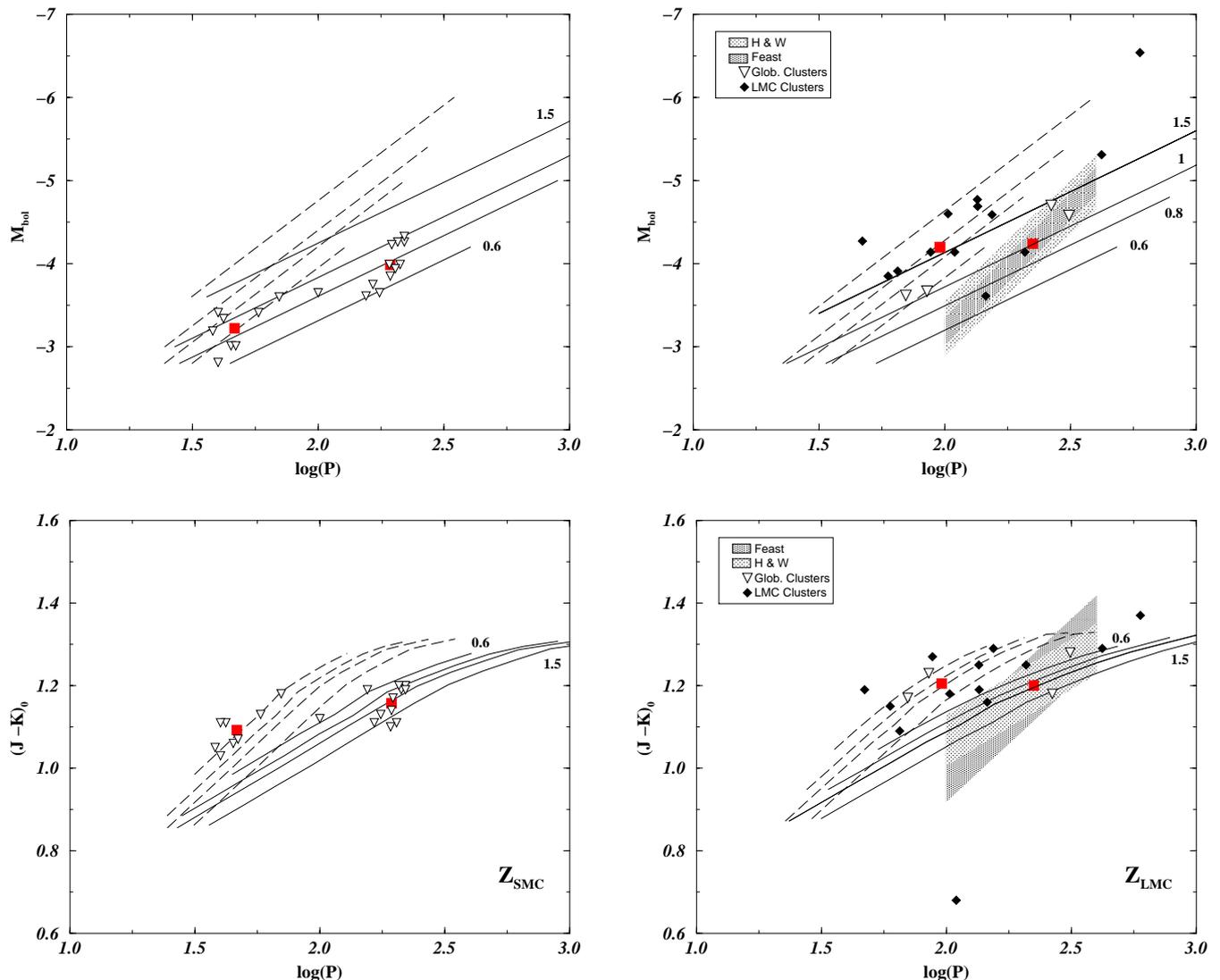
Fig. 5. The PL (*top row*) and PC (*bottom row*) distributions of LPVs in globular and LMC clusters and the ones of LMC Mira-like stars, compared to the calibrated models: fundamental mode (solid lines) and first overtone (dashed lines); filled squares indicate the single-mode barycenters of the data. Left: SMC-like metallicity. Right: LMC-like metallicity. The superimposed linear strips correspond to the whole Mira-like population of the LMC with $P \leq 420$ d: mean PL and PC relations (and the standard deviations about them) given by Feast et al. (1989) and Hughes & Wood (1990)

that occur when the "ellipsoid" crosses the single-mode single-metallicity PLC surfaces with a high incidence.

6.1. Old disk stars

For Group 1, the only possible pulsation mode is the fundamental. The metallicity is $Z = 0.02$ and the mass $0.9\,M_\odot$. On the other hand, Group 2 pulsates on the same mode, but with $Z = 0.027$ and $M = 0.95\,M_\odot$.

A careful look at the PL and LC diagrams on the one hand and the PC on the other, shows a contradiction if a single metallicity is assumed within Group 1. Indeed, when increasing the period and colour, the mass decreases in the latter plane, while it increases in the two former.

This is the 2D translation of the fact that, in 3D, the main symmetry plane of the population is much inclined with respect to the single-Z single-mode theoretical surfaces. If $\Delta \log P$ is supposed to increase by any reasonable amount with the period, the problem is only slightly attenuated. If $\Delta(V-K)$ too is assumed to depend on the period, then it has to reach about -3 at the top of the Group 1 sample. This is at the limit of (or exceeds) the a priori estimates, which anyway exhibit no obvious correlation with the period (see Sect. 4 and Fig. 2). Therefore, complete explanation of the PLC distribution of Group 1 probably requires the metallicity to significantly increase with the period. The mass, too, must strongly increase with the period (even if only the colour shift is invoked). Actually,
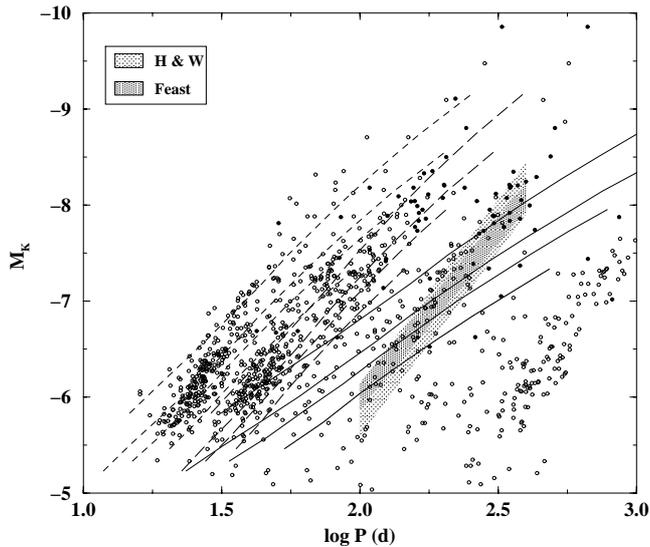
Fig. 6. PL distribution of red variable stars of the LMC in the MACHO data base (magnitudes taken from Wood 1999), compared to the calibrated models: fundamental mode (solid lines), first (long-dashed) and second (dashed) overtones. Also shown are the Mira-like PL relations of Feast et al. (1989) and Hughes & Wood (1990)

if the correction parameters are kept unchanged, a metallicity of 0.04 is found at the barycenter of the Group 1 *sample*, together with a mass of 1.7 $M_\odot$. Concerning the Group 2 sample, the metallicity (0.035) differs just a little from the population mean, and the mass is 1.2 $M_\odot$.

6.2. Thin-disk stars

Group 3 stars cannot pulsate in the fundamental mode, since this would require $Z = 0.44$. If we assume that they are pulsating on the first overtone, the mean mass and metallicity are 1.25 $M_\odot$ and $Z = 0.08$, which seems a bit too metallic for stars of the solar neighbourhood.

However, the MACHO sample (see Fig. 6) suggests that this group may be a mixture of first and second overtone pulsators. That is, the "ellipse" representing the population would, in fact, be a global fit to two steeper, less extended ones. Unfortunately, we were unable to separate these two sub-groups (modes) when doing the luminosity calibration, probably because of the paucity of the sample in view of the observational error bars. We are thus deprived of any direct estimate of the barycenter of each overtone population. Nevertheless, we can use the MACHO sample as a guide to determine two points, typical of the first and second overtone pulsators of the Group 3 population. To this purpose, we first have to shift the MACHO sample by 0.1 magnitude (roughly corresponding to the luminosity and colour shifts involved by the metallicity difference between the LMC and our Galaxy), so that the Group 3 "ellipse" crosses the two bulges at the bottom of the PL strips, and the top-left limit of the Group 3 sample matches that of the MACHO data. Then

the intersections of the strips with the de-biased period–luminosity relation defined by the "ellipse" can be picked out. They are found around $\log P = 1.33$ and 1.85. The corresponding colours are given by the PLC relation found in Paper I. The models fit these points with $Z = 0.04$ and 1.1 $M_\odot$ for the first overtone pulsators, and the same metallicity but 0.75 $M_\odot$ for the second overtone. The latter value is, in fact, a lower boundary of the mean mass, since the adopted point lies at the bottom edge of the second overtone population.

6.3. Extended disk and halo stars

Group 4 stars (Figs. 8 and 11) appear pulsating on the fundamental mode with $Z = 0.009$ and 1.1 $M_\odot$ (0.014 and 1.65 $M_\odot$ for the sample). No solution is found for the first overtone. However, these results must be taken with some caution, because of the paucity of the sample in view of the data error bars (see Sect. 2.1).

7. Discussion

7.1. A problem of mixing length?

A single value of the mixing-length parameter has been used throughout this work. We did not possess enough data on LMC and globular cluster stars to securely calibrate variations of α and of the correction parameters, but some discussion of this deserves attention and is given below.

Comparison between the mixing-length theory and 2D numerical simulations of stellar convection indicate that, at the bottom of the RGB, the mixing-length parameter should exceed the solar value by at least 5% – or even 10% according to some 3D calculations. Low-mass K sub-giant models have $\alpha = \alpha_\odot + 10$–15%, and this parameter is a decreasing function of the effective temperature while it increases with the surface gravity (Freytag & Salaris 1999; Ludwig et al. 1999).

Empirical results are also available: Chieffi & Straniero (1989), Castellani et al. (1991), Bergbusch & Vandenberg (1997), Vandenberg et al. (2000) were able to fit the Red Giant Branch (RGB) of Globular Clusters (masses $\leq 1\ M_\odot$) with the same mixing length parameter as for solar-like stars. However, Stothers & Chin (1995) as well as Keller (1999), showed that the mixing length parameter must exceed $\alpha_\odot$ by roughly 35% at 3 $M_\odot$ and strongly decrease at larger masses (and luminosities) if the red giants and supergiants of Galactic open clusters and of young clusters of the Magellanic Clouds are to be fitted. This non-monotonic behaviour illustrates the competition between the mass and the luminosity in determining the gravity and temperature. No clear metallicity-dependence was found (Stothers & Chin 1996; Keller 1999). Concerning the Horizontal Branch and the Early-AGB, Castellani et al. (1991) found the solar mixing length to be suitable for Globular Cluster stars with masses $\leq 0.8\ M_\odot$.

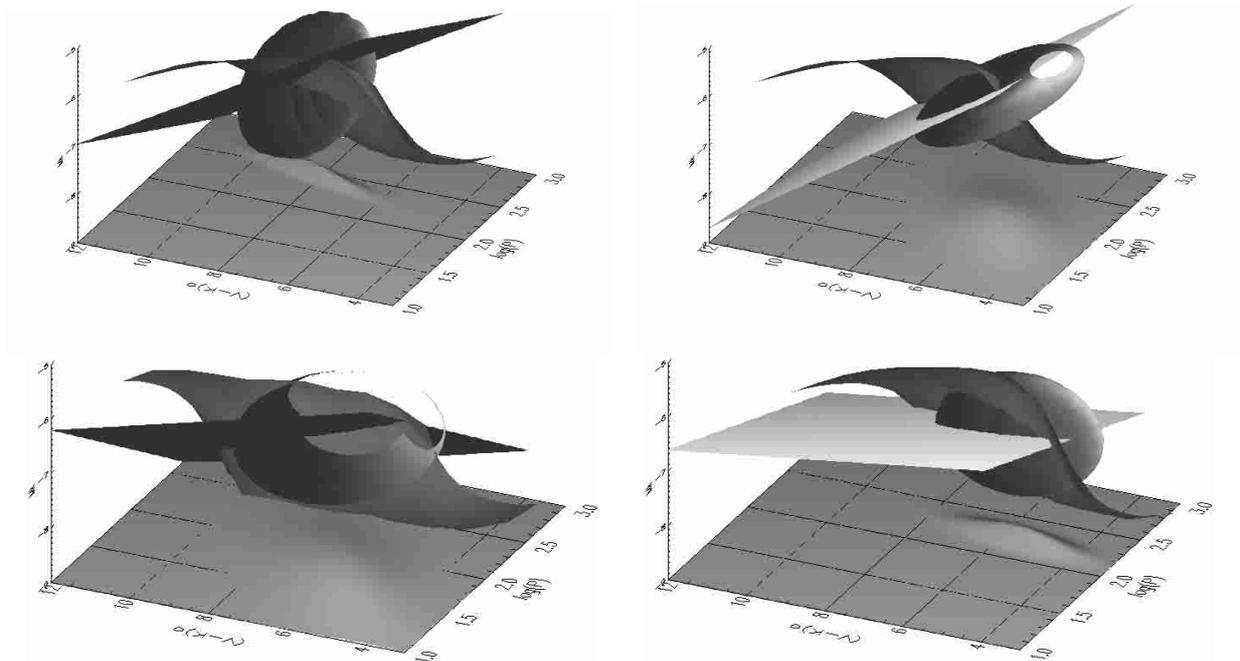Although the LPVs are TP-AGB stars, with much higher and diverse luminosities, these previous works give

Fig. 7. The de-biased PLC distribution of LPVs in the solar neighbourhood (*top row:* Groups 1 and 2; *bottom row:* 3 and 4), compared to calibrated models ($Z = 0.02$, 0.02, 0.04 and 0.01 respectively). Each group appears as a quasi-ellipsoid containing 60% of the population. The PLC relation (main symmetry plane) and the PC distribution (relief on the xy plane) are also represented. The curved surfaces represent the models (fundamental mode for Groups 1, 2 and 4; first overtone for Group 3)
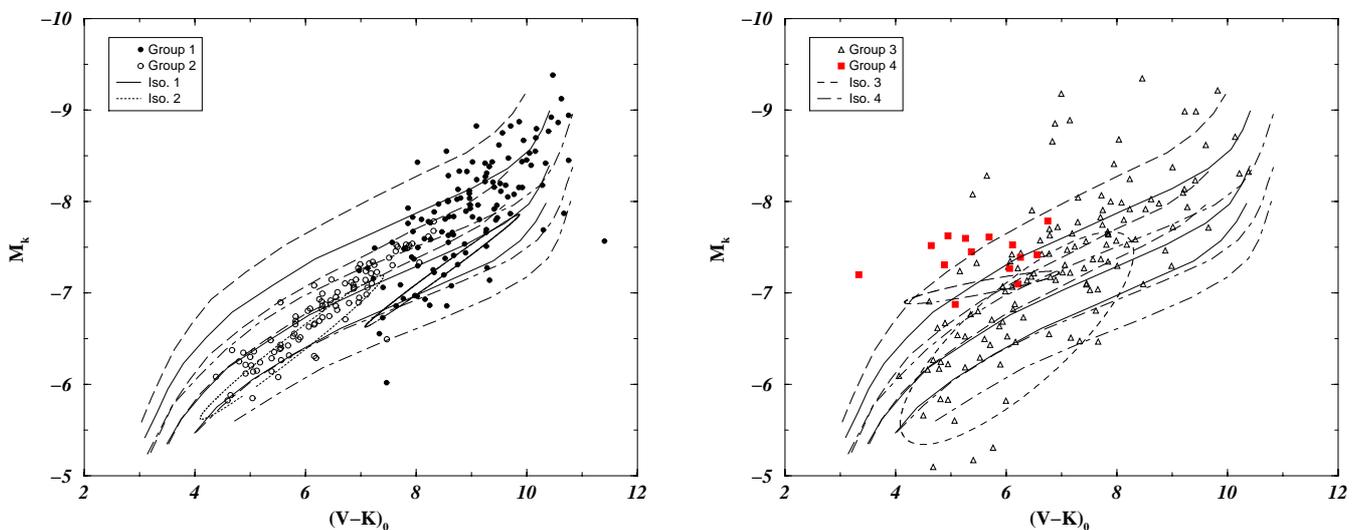

Fig. 8. The LC distribution of local LPVs (sample points and projected, de-biased 2σ iso-probability contours of the populations), compared to calibrated models (solid lines: $Z = 0.02$; dashed: $Z = 0.01$; dot-dashed: $Z = 0.04$; masses are 0.8, 1 and 1.5 $M_\odot$ from the bottom to the top)

us the order of magnitude of the possible variations of α between the mean mass or luminosity that were the basis of the calibration (barycenter of the LMC Mira-like stars) and 3 $M_\odot$ or the maximum luminosity of the sample. We have thus derived a mass–mixing-length relation from the RGB results, by performing a spline interpolation and scaling so as to match the α and mass that were obtained at the barycenter of the LMC Mira-like stars.

On the other hand, when investigating the possibility of a luminosity-dependence, we have assumed that α varies by 35% every 2 magnitudes.

If α depends on the mass, the isomass lines in this paper must move away from each other in the PL and LC diagrams below 3 $M_\odot$ and get closer again at larger masses. This means that, in the previous sections, the masses that appeared larger than 1 $M_\odot$ and the corresponding
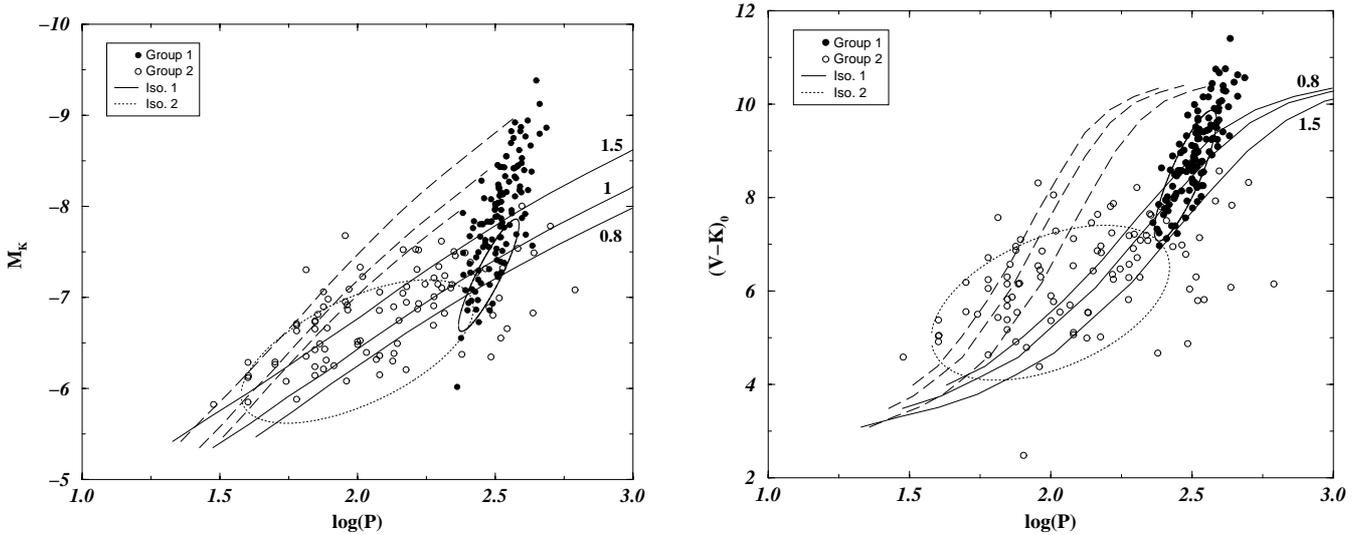
Fig. 9. The PL **a)** and PC **b)** distributions of old-disk LPVs, compared to calibrated models with $Z = 0.02$: fundamental mode (solid lines) and first overtone (dashed lines)
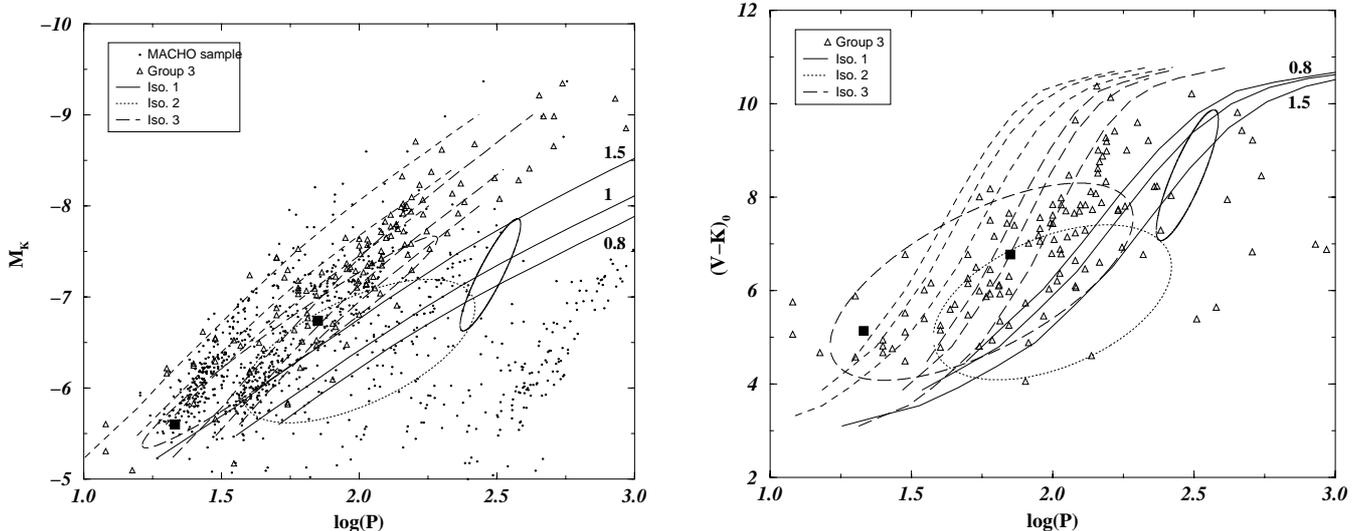

Fig. 10. The PL **a)** and PC **b)** distributions of thin-disk LPVs, compared to calibrated models with $Z = 0.04$: fundamental mode (solid lines), first overtone (long-dashed lines) and second overtone (dashed lines). Also shown are the de-biased distributions of old-disk stars and, shifted by $+0.1$ mag, the MACHO sample (dots). The two squares show the adopted typical first and second overtone pulsators in the thin-disk population

metallicities were respectively over- and underestimated. As a consequence, a totally unlikely metallicity ($Z > 0.10$) is now required in a large part of Group 1. The Group 4 sample, too, gets unlikely high mass and metallicity in view of its kinematics. Moreover, the period corrections also have to be increased, which makes the model fit more unlikely in view of the theoretical background. The mixing-length parameter is thus probably not or is little dependent on the mass. The same arguments also allow us to rule out the hypothesis that it increase with the luminosity.

On the other hand, let us assume that α is a decreasing function of L. Then, the slopes of the isomass lines get smaller in the PL and LC diagrams. The same

combination of $\Delta \log P_0$, $\Delta(J - K)$ and $\alpha_{\mathrm{LMC\ mira}}$ as in the preceding sections gives a better fit, with a mass discrepancy $\delta M = 0.04\ M_\odot$ for the SMC-like GC fundamental pulsators. However, we get $\delta M = 0.08\ M_\odot$ for the first overtone pulsators. As a compromise, we may adopt $\Delta \log P_0 = +0.126$, $\Delta(J - K) = -0.03$ and $\alpha_{\mathrm{LMC\ mira}} = 2.22$, yielding $\Delta \log P_1 = 0.033$ and $\delta M = 0.07\ M_\odot$ for both modes. Then, the deviation of $\Delta(J - K)$ from the mean of its a priori estimates (see Sect. 4.4), expressed in units of standard deviation, defines a scaling to apply to $\Delta(V - K)$, which yields -0.53.

As a first consequence, this hypothesis helps to solve a puzzling contradiction that we found in Figs. 4 and 5: while the theoretical mass increases along the Mira-like
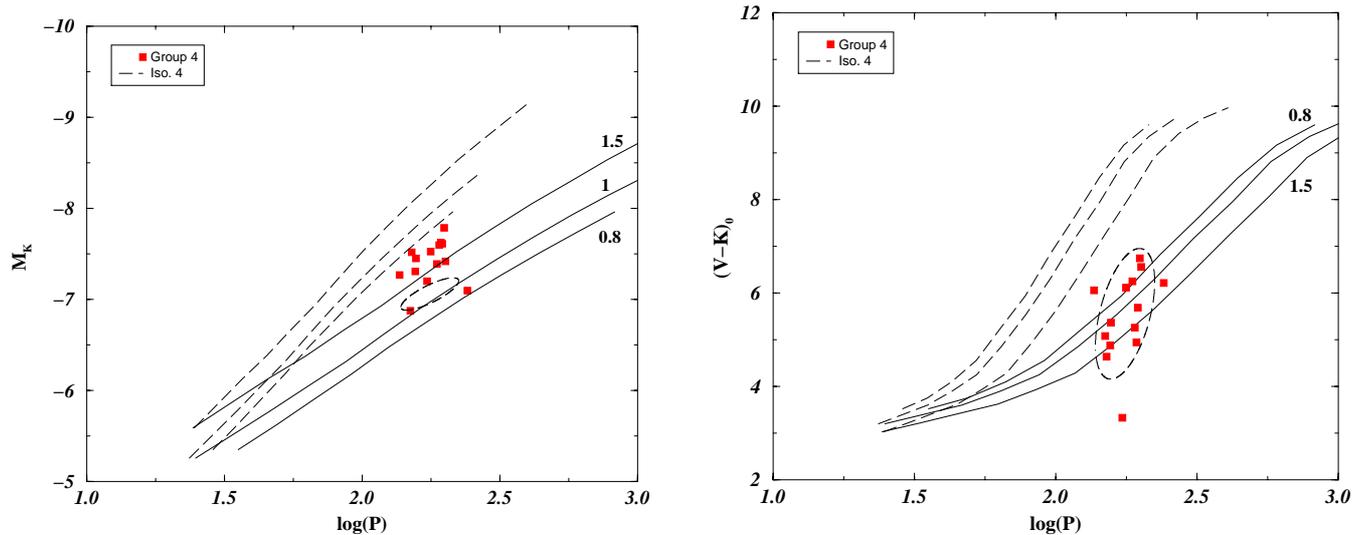
Fig. 11. The PL **a)** and PC **b)** distributions of extended-disk/halo LPVs, compared to calibrated models with $Z = 0.01$

strip of the LMC in the PL and PC planes it decreases in the PC diagram if a constant mixing length is assumed. A bit of metallicity dispersion and of period-dependence of the correction parameters might also help.

The barycenter masses and metallicities of the local, LMC and GC populations are shown in Table 1, together with the ones obtained with a constant α. One can see that $<M>$ and $<Z>$ are nearly constant for Groups 1 and 4. For Groups 2 and 3 (1st ov.), the mean masses decrease by 10%, but the metallicities reach high values that are unlikely for old stars of the solar neighbourhood. Moreover, no solution is found for the Group 3 second-overtone pulsators, even when varying $\Delta \log P_2$ by a factor 3. All of this, together with the very low mass found for GC 1st overtone pulsators, suggests that α is actually less luminosity-dependent than the adopted rate[3].

If we further increase the luminosity-dependence of the mixing length, smaller or even null period corrections of the fundamental and first overtone modes can be adopted. For example, if α varies by 70% every 2 magnitudes, the compromise is reached at $\Delta \log P_0 = 0.063$, $\Delta(J-K) = -0.065$ and $\alpha_{\mathrm{LMC\ mira}} = 2.09$, yielding $\Delta \log P_1 = -0.004$. However, the fit is not better ($\delta M = 0.07\ M_\odot$ for both modes) and the mean mass of the SMC-like GC first-overtone pulsators becomes definitely unlikely (0.4 $M_\odot$).

Concluding, the mixing-length parameter probably decreases along the AGB, but its variation should not exceed 15% per magnitude. Clearly *positive* period corrections are anyway required, especially for the fundamental mode ($> 30\%$) but also for the first overtone ($> 8\%$) and the latter relative shift is always smaller than a third of the former.

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[3] New calculations performed while allowing $\Delta \log P$ to vary with the luminosity have shown that α can not decrease by more than 8% per magnitude along the AGB if the mean mass of the 1st overtone pulsators is required to be $\geq 0.5\ M_\odot$.

7.2. Stability, consistency and error bars

Our results form a consistent set including seven or eight populations with five or six different mean metallicities, three pulsation modes and two colour indices. If we vary by 0.1 $M_\odot$ the assumed mean mass of the LMC Mira-like stars, all other masses simply get shifted by a similar amount. The metallicities of the local stars remain unchanged, and the period and colour correction parameters vary by only about 0.05 and 0.015 ($J - K$) respectively.

In the single-α case, the period and colour correction parameters were taken a minima, i.e. a better fit (smaller δM) would have been obtained with larger corrections. As a test, let us increase $\Delta(J-K)$ and $\Delta(V-K)$ by one standard deviation of the a priori estimates (Fig. 2), this corresponds to $\Delta \log P = 0.24$ for the fundamental mode and 0.101 for the first overtone. Then, the mean masses of Groups 1 through 3 increase by less than 2% and the metallicities by only 0.002. This is of course negligible. Thus, our results are stable, which is confirmed by their similarity when a luminosity-dependent mixing length is adopted (see Table 1).

On the other hand, the error bars of the periods, magnitudes and colours at the barycenter of each population (LMC, Globular Clusters and solar neighbourhood) are small (see Sects. 2 and 5) and yield uncertainties of a few percent on masses and, for the solar neighbourhood, 5–15% on metallicities (except perhaps for Group 4).

Since any change of the adopted temperature scale would automatically translate into the colour correction parameters (the calibrated values as well as the a priori estimates shown in Fig. 2) and into the luminosity dependence of α, the uncertainty of this relation does not significantly affect our results at constant Z. However, metallicity differences with respect to the LMC might be a little overestimated, if molecular opacities were significantly underestimated in the atmospheric models.

Support for this model may be seen in the consistency of the mean masses and metallicities of the local populations of LPVs with their respective kinematics, determined in Paper I, and their similar evolutionary stages:

Group 1, which has the kinematics of old disk stars and is mostly composed of Miras, is actually found having about the solar metallicity, just as usually assumed.

Concerning Group 3, first and second overtone pulsators have the same, higher mean metallicity, consistent with the thin-disk kinematics.

Also consistently with its kinematics (extended disk and halo), Group 4 has a lower metallicity than the others.

Last, compared to Group 1, Group 2 is found to be much more metallic and a bit less massive (L-dependent α) or a little more metallic and massive (single α). Since these stars are slightly less evolved and may lose 0.05 $M_\odot$ in roughly 10^5 years, the mean initial mass must be similar to or smaller than that of Group 1. So, in both cases, Group 2 stars must be a little older (Vassiliadis & Wood 1993). This is consistent with the larger velocity dispersion and scale height found in Paper I. The evolutionary aspects of this work will be further investigated in a forthcoming paper.

7.3. Theoretical interpretation

The significant, positive period correction that has to be applied to linear models contradicts the hydrodynamical calculations. Indeed, we have seen in Sect. 4.2 that our $\Delta \log P_0$ would range from -0.19 to -0.03 if the models of Ya'ari & Tuchman (1996, 1998) did represent real stars. This would lead to large mass discrepancies ($0.1 < \delta M \leq 0.35$ $M_\odot$) for the fundamental pulsators of the SMC-like clusters, even with a luminosity-dependent mixing length. Moreover, the corresponding colour corrections would be extreme values among the a priori estimates calculated in Sect. 4: $-0.15 \leq \Delta(J-K) \leq -0.10$ and $-2.7 \leq \Delta(V-K) \leq -1.9$. This is quite a bit for an average behaviour!

Another interesting – though weaker – argument is derived from evolutionary considerations. For the predicted period shifts, the mean metallicity and mass ratios of Group 1 to Group 2 are $8 \geq Z_1/Z_2 > 0.8$ and $1.5 \geq M_1/M_2 \geq 1.1$. Detailed evolutionary calculation is beyond the scope of this paper but if the periods predicted by Ya'ari & Tuchman were right, Group 2 would probably be a little younger than Group 1, in contradiction with the kinematics.

Summarizing, the nonlinear behaviour calculated by Ya'ari & Tuchman (1996) is unlikely in view of the available data. Let us try to explain this:

As stated in Sect. 4.1, the core mass–luminosity relation assumed in our calculations holds over a limited part of the thermal pulsation cycle, but the effect on the period and temperature is very small and the opposite sign of the calibrated period and colour correction parameters.

The same section also shows that phase-lagged convection, horizontal opacity averaging and turbulent pressure all together might yield a period increase by as much as 40% for the fundamental and 25% for the first overtone, the relative shift of the latter always being larger than a third of the former. In case of negligible nonlinear effects, this could explain the correction parameters of the fundamental or the first overtone but not both, since the ratio would be wrong[4]. Furthermore, the nonlinear effects predicted by Ya'ari & Tuchman would strongly reduce the final period of the fundamental while not significantly changing the first overtone, so that both modes would finally exhibit similar shifts with respect to our LNA models.

In other terms, if an improved physics of the subphotospheric regions manages to explain the observed first overtone, then there must remain a significant, positive shift of the actual fundamental mode ($\gtrsim 15\%$, possibly much more) with respect to its theoretical period. The only explanation seems to be the coupling of the stellar envelope with the circumstellar layers and the subsequent wind, evoked in Sect. 4.3 above.

8. Conclusion

The aim of this paper was to interpret the results of Barthès et al. (1999) in terms of pulsation modes and fundamental parameters, i.e. the fact that the Long-Period Variable stars of the solar neighbourhood are distributed among four groups (according to kinematic and photometric criteria), and to study the period–luminosity–colour distributions of these groups. This was done by confronting them with a grid of linear nonadiabatic pulsation models.

Preliminary discussion of the colour–temperature relations and of the existing linear and nonlinear modelling codes showed that the periods and colours predicted by LNA models may significantly differ from the observed values, even if the adopted fundamental parameters are the true ones. In order to mimic this behaviour, we added a few free parameters to the LNA models: a systematic period correction $\Delta \log P$ for each mode, and systematic colour corrections $\Delta(V-K)$ and $\Delta(J-K)$. These parameters, as well as the mixing length, were then calibrated by demanding that consistent masses be derived from the period and from the colour, for the fundamental and first-overtone pulsators of the LMC and of globular clusters with LMC or SMC metallicity. It was assumed that the mean mass of the Mira-like stars (fundamental pulsators) of the LMC is 1 $M_\odot$. Then, the mean mass of the LMC first-overtone pulsators was found to be about 0.95 $M_\odot$, while that of the fundamental and first-overtone pulsators with SMC metallicity was 0.8 and 0.6 $M_\odot$ respectively (or down to 0.75 and 0.5 $M_\odot$ if the mixing-length parameter is allowed to strongly vary with the luminosity).

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[4] The same remark holds a fortiori for the calculations based on Wood's code, which give both modes about the same shift.

We were thus able to determine the pulsating mode and the mean masses and metallicities of the neighbouring LPV populations:

- Group 1 stars (old disk LPVs, mostly Miras) are pulsating on the fundamental mode. The mean mass is $< M > \simeq 0.9\,M_\odot$ and the mean metallicity $< Z > \simeq 0.02$, both strongly increasing with the period;
- Group 2 stars (old disk LPVs, mostly SRb's) are pulsating on the fundamental mode also, with $< M > \simeq 0.9\,M_\odot$ and $< Z > \gtrsim 0.03$. Metallicity variations of about 50% are likely;
- Group 3 stars (thin-disk LPVs, mostly SRb's) are pulsating on the first and second overtones, with $< M_{1ov} > \approx 1.05\,M_\odot$, $< M_{2ov} > > 0.75\,M_\odot$ and $< Z > \geq 0.04$;
- Group 4 stars (extended disk and halo) appear to be pulsating on the fundamental mode with $< M > \simeq 1.1\,M_\odot$ and $Z \simeq 0.01$. This result must however be taken with caution.

These findings, based on the $V - K$ index as a temperature estimator, have been checked and confirmed by using $J - K$. They were proven to be stable and consistent with the kinematics. The error bars of the masses are about 5%. If one shifts the reference mass of the LMC Miras by any reasonable amount, all other masses are simply similarly shifted and the metallicities remain nearly unchanged. Metallicity differences with respect to the LMC may be a little overestimated if molecular opacities have been significantly underestimated in the atmospheric models. Nevertheless, the hierarchy holds anyway.

The mixing-length parameter probably decreases along the AGB, but its variation should not exceed 15% per magnitude. This was taken into account in the abovementioned results.

This study confirms the findings in Paper I, the discrimination between Miras and semiregulars is not pertinent: Groups 1 and 2 not only have similar kinematics but also the same pulsation mode.

It has also been shown that both the linear and nonlinear models that were the basis of all previous studies of LPV pulsation are probably far from the real pulsational behaviour of these stars. While dynamical calculations including a modern equation of state predict a strong reduction of the fundamental nonlinear period with respect to the linear one, important, *positive* systematic corrections have to be applied to the periods of our linear models (30–45% for the fundamental mode and 8–13% for the first overtone). Improvements of the physics of the sub-photospheric envelope (phase lagged convection, turbulent pressure, horizontal opacity averaging...) appear insufficient to explain these two shifts altogether, so that the actual fundamental period should always exceed the theoretical one by at least 15%. This led us to conclude that all existing linear and nonlinear pulsation codes probably suffer from neglecting the stellar wind generated by the interaction with the circumstellar envelope.

As a consequence, the works of Barthès & Tuchman (1994) and Barthès & Mattei (1997), who confronted LNA models with the Fourier components of the lightcurves of a few nearby Miras and concluded in favour of the first overtone, should now be reconsidered by taking into account the necessary period corrections and the variations of metallicity and, possibly, mixing-length parameter.

This study may also have consequences in AGB evolutionary calculations, especially those that use the period as a substitute for a fundamental parameter (M or $T_{\rm eff}$) or as the variable in the empirical mass-loss function (e.g. Whitelock 1986; Vassiliadis & Wood 1993; Reid et al. 1995; Marigo et al. 1996). Indeed, these studies are based on pulsation models by Wood that appear to strongly overestimate the periods and their dependence on luminosity and metallicity, probably because of the equation of state. Concerning the fundamental mode, this peculiarity allows Wood's models to roughly mimic the abovementioned systematic period shifts and variation of the mixing length parameter, but with an uncertainty that remains to be assessed. Moreover, while many Long Period Variables are obviously pulsating on the first or second overtone, this possibility is usually neglected in evolutionary calculations, and Wood's models appear definitely inappropriate for these modes. These issues will be investigated in a forthcoming paper.

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