On the duration of the subsonic propeller state of neutron stars in wind-fed mass-exchange close binary systems

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Abstract. The condition for the *subsonic propeller → accretor* state transition of neutron stars in wind-fed mass-exchange binary systems is discussed. I show that the value of the break period, at which the neutron star change its state to *accretor*, presented by Davies & Pringle (1981) is underestimated by a factor of 7.5. The correct value is $P_{\rm br} \simeq 450 \, \mu_{30}^{16/21} \, \dot{M}_{15}^{-5/7} \, (M/M_\odot)^{-4/21}$ s. This result forced us to reconsider some basic conclusions on the efficiency of the propeller spindown mechanism.

Key words. accretion – propeller spindown – stars: close binaries – stars: neutron star

1. Introduction

The sequence of states which a magnetized neutron star in a wind-fed mass-exchange binary system follows as it spins down from the initially very short periods can be expressed in the form of the following chain: *ejector → propeller → accretor*. This classification, first suggested by Shvartsman (1970), reflects three different evolutionary stages and three different mechanisms of energy release responsible for the neutron star emission.

The spindown of a neutron star in the state of ejector is governed by the canonical spin-powered pulsar mechanism. The spindown power dominates the star energy budget and is spent predominantly to the generation of magneto-dipole waves and particle acceleration. The pulsar-like spindown ceases when the pressure of the material being ejected by the neutron star can no longer balance the pressure of the surrounding gas. The stellar wind plasma penetrates into the neutron star light cylinder and interacts with the star magnetosphere. This corresponds to the neutron star state transition: *ejector → propeller*.

Neutron star in the state of propeller is spinning down due to the interaction between its fast rotating magnetosphere and the surrounding material. Davies et al. (1979) and Davies & Pringle (1981) have shown that during this state the star magnetosphere is surrounded by a spherical quasi-static envelope in which the plasma temperature is of the order of the free-fall temperature,

$$T(R) \simeq T_{\rm ff}(R) = \frac{GM_{\rm ns} m_{\rm p}}{kR}. \qquad (1)$$

Here $M_{\rm ns}$ is the mass of the neutron star, $m_{\rm p}$ is the proton mass and k is the Boltzmann constant. The rotational energy loss by the neutron star is convected up through the envelope by the turbulent motions and lost through its outer boundary. The structure of the envelope and the spindown rate of the neutron star depend on the value of the ratio:

$$\kappa = \frac{\omega R_{\rm m}}{V_{\rm s}(R_{\rm m})},$$

where $R_{\rm m}$ is the magnetospheric radius, $\omega = 2\pi/P_{\rm s}$ is the neutron star angular velocity and $V_{\rm s}(R_{\rm m})$ is the sound speed at the magnetospheric radius, which according to Eq. (1) is of the order of the free-fall velocity, $V_{\rm ff}$:

$$V_{\rm s}(R_{\rm m}) \simeq V_{\rm ff}(R_{\rm m}) = \sqrt{\frac{2GM_{\rm ns}}{R_{\rm m}}}. \qquad (2)$$

On this basis Davies et al. (1979) distinguished three sub-states of the propeller state[1]: (b) very rapid rotator ($\kappa \gg 1$); (c) supersonic propeller ($\kappa \gtrsim 1$) and (d) subsonic propeller ($\kappa < 1$).

In cases "b" and "c" the magnetospheric radius of the neutron star exceeds its corotational radius,

$$R_{\rm cor} = \left(\frac{GM_{\rm ns}}{\omega^2} \right)^{1/3}.$$

This means that the neutron star in these cases is in the centrifugal inhibition regime and hence, the effective accretion onto its surface is impossible.

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[1] According to their classification case "a" corresponds to the pulsar-like spindown, i.e. the ejector state.

In the case "d" the magnetospheric radius is smaller than the corotational radius. In this situation the plasma being penetrated from the base of the envelope into the star magnetic field is able to flow along the magnetic field lines and to accrete onto the star surface. However the effective plasma penetration into the magnetosphere does not occur, if the magnetospheric boundary is interchange stable. According to Arons & Lea (1976) and Elsner & Lamb (1976) the onset condition for the interchange instability of the magnetospheric boundary reads

$$T < T_{\mathrm{cr}} \simeq 0.3 \ T_{\mathrm{ff}}.$$

This means that the neutron star can change its state from the *subsonic propeller* to *accretor* only when the cooling of the envelope plasma (due to the radiation and convective motions) dominates the energy input to the envelope due to the propeller action by the neutron star.

Investigating this particular situation Davies & Pringle (1981, hereafter DP81) have shown that the energy input to the envelope dominates the energy losses until the spin period of the star reaches the break period, P_{br}. Assuming the following values of the neutron star parameters: the magnetic moment $\mu = 10^{30} \mu_{30} \ \mathrm{G \, cm^3}$ and the mass $m = 1(M_{\mathrm{ns}}/M_{\odot})$, and putting the strength of the stellar wind (in terms of the maximum possible accretion rate) $\dot{M}_0 = 10^{15} \ M_{15} \, \mathrm{g \, s^{-1}}$ they estimated the value of the break period as $60 \, \mathrm{s}$.

However, putting the same values of parameters and following the same method of calculations I found the value of P_{br} to be of the order of $450 \, \mathrm{s}$, i.e. by a factor of 7.5 larger than that previously estimated in DP81. In this letter I present the calculations and show that this result forced us to change some basic conclusions about the origin of the long periods X-ray pulsars.

2. Break period

According to the picture presented by Davies & Pringle the magnetosphere of the neutron star in the state of subsonic propeller is surrounded by the adiabatic ($p \propto R^{-5/2}$) spherically symmetrical plasma envelope. Until the energy input to the envelope dominates the energy losses the temperature of the envelope plasma is of the order of the free-fall temperature, $T \simeq T_{\mathrm{ff}}$ and, correspondingly, the sound speed is of the order of the free-fall velocity, $V_{\mathrm{s}} \simeq V_{\mathrm{ff}}$. Under this condition the height of the homogeneous atmosphere through out the envelope is comparable to R and thus, the envelope is extended from the magnetospheric radius,

$$R_{\mathrm{m}} \simeq r_{\mathrm{m}} \equiv \left(\frac{\mu^2}{\dot{M}_0 \sqrt{2GM_{\mathrm{ns}}}} \right)^{2/7}$$
$$= 1.2 \ 10^9 \, \mathrm{cm} \ \mu_{30}^{4/7} \ \dot{M}_{15}^{-2/7} \ m^{-1/7}, \qquad (3)$$

up to the accretion radius of the neutron star,

$$R_{\alpha} \equiv \frac{2GM_{\mathrm{ns}}}{V_{\mathrm{rel}}^2} = 2.7 \ 10^{10} \, \mathrm{cm} \ m \ V_8^{-2}.$$

Here, V_8 is the relative velocity between the neutron star and the stellar wind plasma, V_{rel}, expressed in units of $10^8 \, \mathrm{cm \, s^{-1}}$ and $\dot{M}_0$ is strength of the stellar wind which is expressed following DP81 in terms of the maximum possible accretion rate:

$$\dot{M}_0 = \pi R_{\alpha}^2 \rho_{\infty} V_{\mathrm{rel}},$$

where ρ_{∞} is the density of the stellar wind plasma. Within the considered picture the envelope is quasi-static, i.e. the mass flux through the envelope is almost zero. In this situation the physical meaning of the parameter $\dot{M}_0$ is the rate by which the stellar wind plasma overflows the outer edge of the envelope compressing the envelope plasma.

The interaction between the fast rotating magnetosphere and the base of the envelope leads to the turbulization of the envelope plasma. The velocity of the convection motions at the magnetospheric boundary is obviously limited as

$$V_{\mathrm{t}} \lesssim \omega R_{\mathrm{m}}. \qquad (4)$$

Under the condition $R_{\mathrm{m}} < R_{\mathrm{cor}}$ the maximum velocity of the convective motions is smaller than V_{s} and hence, the Mach number at the base of the envelope is $M_{\mathrm{Mach}} = V_{\mathrm{t}}/V_{\mathrm{s}} < 1$.

The rate of energy loss by the neutron star and, correspondingly, the energy input to the envelope in this case can be expressed as (see Eqs. (3.2.1) and (3.2.2) in DP81)

$$L_{\mathrm{d}} = 2\pi R_{\mathrm{m}}^2 \rho V_{\mathrm{t}}^3.$$

As it has been argued by Davies & Pringle the cooling of the envelope plasma is determined by the combination of convective motions and bremsstralung radiation. In order to evaluate the energy balance between the heating and cooling processes they introduced the convective efficiency parameter (see also Cox & Guili 1968):

$$\Gamma = \frac{\text{Excess heat content of convective blob}}{\text{Energy radiated in the lifetime of a blob}}.$$

Under the conditions of interest this parameter can be expressed as (for discussion see DP81, p. 221)

$$\Gamma = M_{\mathrm{Mach}}^2 \left[\frac{V_{\mathrm{t}} t_{\mathrm{br}}}{R} \right] = \frac{V_{\mathrm{t}}^3 t_{\mathrm{br}}}{V_{\mathrm{s}}^2 R}, \qquad (5)$$

where t_{br} is the bremsstralung cooling time:

$$t_{\mathrm{br}} = 6.3 \ 10^4 \left[\frac{T}{10^9 \, \mathrm{K}} \right]^{1/2} \left[\frac{n}{10^{11} \, \mathrm{cm^{-3}}} \right]^{-1} \mathrm{s}. \qquad (6)$$

Here, n is the number density of the envelope plasma which at the base of the envelope can be evaluated as

$$n(R_{\mathrm{m}}) = \frac{\mu^2}{4\pi k T_{\mathrm{ff}}(R_{\mathrm{m}}) R_{\mathrm{m}}^6}. \qquad (7)$$

As it has been shown in DP81 the cooling of the envelope during the subsonic propeller state occurs first at its inner radius. Thus, the energy input to the envelope due to

the neutron star propeller action dominates the radiative losses if $\Gamma(R_{\rm m}) \gtrsim 1$.

Combining Eqs. (1–7) I find the condition $\Gamma(R_{\rm m}) \gtrsim 1$ to be satisfied if the spin period of the neutron star is $P_{\rm s} \lesssim P_{\rm br}$, where the break period is

$$P_{\rm br} \simeq 450 \ \mu_{30}^{16/21} \ \dot{M}_{15}^{-5/7} \ m^{-4/21} \ {\rm s}. \qquad (8)$$

This value of the break period exceeds the value of $P_{\rm br}$ presented in DP81 by a factor of 7.5 (see Eq. (4.8) in DP81). Hence the natural question about the reason of this inconsistency arises. One of the most possible reasons is that Davies & Pringle have mistakenly used the value of the magnetospheric radius: $4.4 \ 10^8 \mu_{30}^{4/7} \dot{M}_{15}^{-2/7} m^{-1/7}$ cm (see Eq. (3.2.3) of their paper) instead of the correct value which is expressed in their paper by Eq. (2.5), i.e. $10^9 \mu_{30}^{4/7} \dot{M}_{15}^{-2/7} m^{-1/7}$ cm. Taking into account that $P_{\rm br}(R_{\rm m}) \propto R_{\rm m}^{5/2}$ one finds that the correct value of $P_{\rm br}$ should be larger than that derived in DP81 by a factor of 7.7, i.e. very close to the value of the break period obtained in this letter.

3. Discussion

One of the main astrophysical reasons for the investigation of the spindown of neutron stars is the existence of X-ray sources which display pulses with long periods (in excess of 100 s). On the basis of their calculations Davies & Pringle suggested that the periods of neutron stars spinning down due to propeller mechanism can be as long as 100 s only if the stars are situated in the weak stellar wind, i.e. $\dot{M}_0 < 4 \ 10^{14}$ g s^{-1}. They also pointed out that in this case however it is difficult to account for a substantial population of long period pulsators.

In the light of the recalculated value of the break period obtained in this paper (Eq. (8)) I find that the propeller mechanism can be responsible for the long spin period of a neutron star even if it is situated in the essentially stronger stellar wind:

$$\dot{M}_0 \lesssim 8 \ 10^{15} \ \mu_{30}^{16/15} \ m^{-4/15} \ P_{100}^{-7/5} {\rm g \, s}^{-1}, \qquad (9)$$

where P_{100} is the observed spin period of the neutron star expressed in units of 100 s. The corresponding spindown time-scale of the neutron star in the state of subsonic propeller is

$$\tau_{\rm d} \simeq 10^5 \ \mu_{30}^{-2} \ m \ I_{45} \ P_{100} \ {\rm yr}, \qquad (10)$$

i.e. smaller that the characteristic evolutionary time-scale of the early spectral type supergiants. Here I_{45} is the moment of inertia of the neutron star expressed in units of 10^{45} g cm^2.

4. Conclusion

The value of the break period at which the spinning down neutron star changes its state from the *subsonic propeller* to *accretor* obtained by Davies & Pringle (1981) is underestimated by a factor of 7.5. The incorporation of the re-estimated value of the break period into the spindown scenario suggested by Davies & Pringle shows that the propeller mechanism can be responsible for the origin of the long period X-ray pulsators even if the strength of the stellar wind, in which a neutron star is situated, is in excess of 10^{15} g s^{-1}. The analysis of the spin evolution of a neutron star situated in the strong stellar wind will be presented in a forthcoming paper.

References

Arons, J., & Lea, S. M. 1976, ApJ, 207, 914
Davies, R. E., Fabian, A. C., & Pringle, J. E. 1979, MNRAS, 186, 779
Davies, R. E., & Pringle, J. E. 1981, MNRAS, 196, 209 (DP81)
Elsner, R. F., & Lamb, F. K. 1976, Nature, 262, 356
Cox, J. P., & Guili, R. T. 1968, Principles of Stellar Structure (Gordon & Breach, New York)
Shvartsman, V. F. 1970, Radiofizika, 13, 1852