

# The orbit evolution of 32 plutinos over 100 million year

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**Abstract.** The orbits of thirty two plutinos that are presently in the 3:2 mean motion resonance with Neptune have been integrated numerically and accurately to  $10^8$  years into the future. Fourteen of them are found in unstable orbits after encountering Neptune or Pluto. Six of eighteen plutinos with stable orbits are in the Kozai resonance or around its separatrix zone. No node to node, perihelion to perihelion secular resonance or the so called 1:1 super resonance are found.

**Key words.** Kuiper belt, Oort cloud – minor planets, asteroids

## 1. Introduction

Edgeworth (1949) and Kuiper (1951) independently suggested that there would exist a large number of minor bodies in the solar system beyond the orbit of Neptune, which are the remains of planetesimals during the period of formation of the solar system. Interest in their conjecture has increased greatly since the first object was discovered in this region by Jewitt & Luu (1993) in 1992. Astronomers believe that these bodies would give us a better understanding of the origin and evolution of the solar system. This region is now known as the Kuiper belt. Fernandez (1980), Duncan et al. (1988) numerically simulated the orbital motion of Kuiper belt objects (KBOs) and pointed out that they might be a source of the Jupiter-family short period comets. Duncan et al. (1995) explored their orbital stability. They numerically integrated the orbits of thousands of KBOs up to the age of the solar system. They found that the orbits of KBOs with very small eccentricities,  $e$ , and with semi-major axes,  $a$ , larger than 36 AU are mostly stable with the  $40 < a < 42$  AU region being exceptional due to the presence of secular resonances. They pointed out that for KBOs with larger  $e$  and  $35 < a < 42$  AU the low order mean motion resonances with Neptune play a key role on their orbital stability (for an extensive review and comment, see also Malhotra et al. 2000). Up to now nearly 100 observed KBOs have been observed at multiple oppositions and are presumed to have orbits that are reasonably reliable. For  $a < 42$  AU, they are trapped into mean motion resonances and have large eccentricities; for  $a > 42$  AU, their positions are not particularly

related to mean motion resonances and usually have small eccentricities. Of reliably determined orbits, over 30% are trapped in the 3:2 mean motion resonance and they are the so called plutinos (Jewitt et al. 1996). The dynamic structure of the 3:2 resonance has been explored extensively (for example, Morbidelli 1997) since Cohen & Hubbard (1965) found that Pluto is in the 3:2 mean motion resonance with Neptune. Malhotra (1993) suggested that Pluto originated from a nearly circular and less inclined orbit and the migration of the major planets in the early stage of the solar system excited its eccentricity and inclination and put it in the 3:2 resonance. Malhotra (1995) pointed out that this theory predicts a large population of plutinos and other resonant KBOs. Yu & Tremaine (1999) adopted a model of the outer planets migration to make a theoretical analysis and a numerical simulation on the stability of plutinos. They found that the orbit of a plutino is stable when its eccentricity is either near that of Pluto ( $|e - e_P|/e_P \lesssim 0.1$ , where  $e$  and  $e_P$  are the eccentricities of the plutino and Pluto respectively) or very different from that of Pluto ( $|e - e_P|/e_P \gtrsim 0.3$ ).

For Pluto, there are two more resonances besides the 3:2 mean motion resonance. They are the Kozai resonance (Williams & Benson 1971), which keeps its perihelion argument,  $\omega$ , librating around  $90^\circ$ , and the so called 1:1 super resonance (Williams & Benson 1971; Milani et al. 1989; Wan et al. 2000), which keeps the libration period of  $\omega$  close to the circulation period of the longitude difference between the ascending nodes of Pluto and Neptune,  $\Omega - \Omega_N$ . Wan et al. (2000) gave the sizes of the stability zones of the three resonances in the space of orbital elements. Every resonance is a kind of protection mechanism to keep Pluto away from Neptune's perturbation.

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**Table 1.** Orbital elements of the five outer planets and the 32 plutinos at 2000.0 in the system of heliocentric invariant plane of 2000.0. An asterisk after a plutino's name marks its orbit based on only one opposition

Object	$a(\text{AU})$	$e$	$i^\circ$	$\omega^\circ$	$\Omega^\circ$	$M^\circ$	Source
Jupiter	5.20421	0.0488	0.3275	58.2257	317.3093	18.8574	DE234
Saturn	9.58195	0.0557	0.9296	325.7585	123.8796	320.3602	DE234
Uranus	19.22950	0.0444	1.0292	218.3694	312.2091	142.9217	DE234
Neptune	30.10557	0.0112	0.7244	202.1179	194.9391	268.1428	DE234
Pluto	39.26376	0.2447	15.5741	113.5018	110.5503	15.0162	DE234
1993RO	39.87301	0.2100	3.2823	164.3786	195.4980	2.5474	MPC 30688
1993SB	40.12059	0.3303	2.9354	107.1356	325.1308	328.4374	MPC 36882
1993SC	40.12305	0.1915	5.9410	334.9581	340.5007	38.6813	MPC 36882
1994JR1	38.92671	0.1072	2.7134	82.7925	165.4456	4.6545	MPC 34582
1994TB	40.24769	0.3287	13.5215	101.1591	314.1261	333.8047	MPC 36884
1995HM5	38.93880	0.2428	4.7966	42.8043	205.5407	327.3958	MPC 32308
1995QZ9	40.06416	0.1551	19.2446	141.9822	192.4231	31.9877	MPC 30783
1995YY3	39.97136	0.2319	1.1435	130.1496	290.9129	7.0493	MPC 27920
1996RR20	39.94925	0.1745	4.5878	35.5981	180.0305	114.3301	MPC 30785
1996TP66	40.31454	0.3457	7.1108	81.5335	310.6128	359.4938	MPC 36888
1996TQ66	40.10226	0.1367	14.8730	26.4424	4.7160	0.9490	MPC 36888
1997QJ4	40.06144	0.2393	17.3723	84.7594	342.4586	316.8782	MPC 36893
1998HH151	38.44862	0.1546	8.8993	10.7611	204.9529	346.8681	MPC 35440
1998HK151	38.78986	0.2172	5.2893	193.3237	35.7187	3.0587	MPC 34918
1998HQ151	38.75965	0.2721	12.8919	340.4688	234.8507	10.4392	MPC 34919
1998UR43	40.14194	0.2278	7.8891	23.9622	44.7238	341.4225	MPC 36820
1998US43	40.04371	0.1517	11.3229	134.3818	230.9129	34.1429	MPC 36379
1998UU43	39.03955	0.2657	10.4695	257.3069	238.5366	293.6570	MPC 36820
1998VG44	39.84386	0.2584	1.6464	301.8364	147.0948	341.8865	MPC 36821
1998WV24	39.94157	0.1117	1.8881	240.9064	237.3031	299.2427	MPC 36821
1998WZ24*	39.73140	0.1848	3.1866	8.6375	67.0976	1.9796	MPC 35442
1998WU31	40.12269	0.2088	7.6430	130.3469	246.3207	23.0274	MPC 36821
1998WV31	39.82698	0.2721	4.8126	291.5277	44.2743	40.0718	MPC 36821
1998WZ31*	39.82047	0.1698	13.7894	11.5906	45.1206	1.8024	MPC 33766
1999JB132	39.25729	0.2744	13.5829	90.2935	218.2959	296.0702	MPC 35470
1999JC132*	39.25828	0.2441	5.4625	297.7963	19.9211	284.1128	MPC 35470
1999JE132*	39.18590	0.1944	25.5738	264.1790	216.3818	70.5880	MPC 35470
1999JK132*	39.07611	0.1661	15.7368	87.4546	27.2109	80.3354	MPC 35737
1999KR16*	39.08885	0.2274	28.6385	101.5305	208.4649	282.0770	MPC 35737
1999KS16*	39.02739	0.2590	3.6820	320.7394	0.7318	273.9565	MPC 35737
1999TR11*	40.03327	0.2520	15.9936	4.0646	50.3021	0.4365	MPEC 1999-V05
2000FE8*	38.98400	0.0730	6.9935	193.1901	351.4466	0.7180	MPEC 2000-G01

Morbidelli et al. (1995) numerically followed the orbital evolution of five KBOs, including three plutinos, 1993RO, 1993SB and 1993SC. They found that the latter two are orbitally stable but the first one is in chaotic motion and will be expelled from the 3:2 resonance. Five years have passed since their computation. The orbits of these plutinos have been improved and more plutinos have been discovered. The significance of exploring the long term orbital evolution of plutinos is more than just the objects themselves; it also provides a good check on the theoretical dynamic structure of the 3:2 mean motion

resonance. In this article we explore the present and future of plutinos by a long term numerical integration. Our numerical experiments will be described in Sect. 2 and our main results will be reported in Sect. 3.

## 2. Numerical experiments

The model of solar system we adopt consists of the Sun and the five outer planets. The masses of the inner planets have been added to that of the Sun. The masses and the initial heliocentric coordinates and velocities of the

outer planets in the mean equatorial system of 2000.0 are taken from DE234 (Standish 1993, private communication), with epoch June 28, 1969. The sources of the orbit elements of the 32 plutinos are listed in the last column of Table 1. The original elements are in the ecliptic coordinate system of 2000.0 and every plutino has its own epoch. Therefore we first transformed the coordinates and velocities of the outer planets from the mean equatorial system to the ecliptic system, then for each plutino we integrated the outer planets to its specified epoch. Finally, the five outer planets and the 32 plutinos were put into the same epoch 2000.0 and then we transformed the heliocentric ecliptic coordinate system to the heliocentric invariant plane coordinate system. The latter coordinate system consists of the invariant plane of the solar system in which the longitude zero is so chosen that the node of the ecliptic on the invariant plane have the same longitude in the two systems. The integrator for this procedure is a 12th order Cowell prediction-correction algorithm (PECE) with a step size of 10 days.

Table 1 lists the orbital elements of the five outer planets and the 32 plutinos at 2000.0 in the heliocentric invariant plane coordinate system of 2000.0, in which  $a$ ,  $e$ ,  $i$ ,  $\omega$ ,  $\Omega$  and  $M$  are the semi-major axis, eccentricity, inclination, perihelion argument, longitude of node and mean anomaly respectively. An asterisk marks orbits based on only one opposition.

We integrated the whole system for  $10^8$  years starting from 2000.0. In order to ensure the accuracy of this long term integration we use a 12th order symmetric method (Quinlan & Tremaine 1990) as the integrator, which has similar advantages as symplectic integrators. The step size we adopt is 35 days and we save the data every 150 000 steps for further data processing. A  $10^7$  years run by the Cowell PECE integrator was taken as a comparison. This long term integration was run in double precision on a 440 MHz SUN work station. The routine provided by Quinlan applies quadruple precision calculation at key operations in order to reduce the accumulation of roundoff errors.

### 3. Main results

We have found that the orbit evolution of the 32 plutinos can be classified into five categories: (a) The plutino is eventually expelled from the 3:2 resonance and falls inside the Hill radius of Neptune or Pluto. (b) The critical argument of the 3:2 mean motion resonance,  $\sigma = 3\lambda - 2\lambda_N - \varpi$ , switches between libration and circulation, where  $\lambda_N$  and  $\lambda$  are the mean longitudes of Neptune and the plutino respectively and  $\varpi$  is the longitude of the perihelion of the plutino. (c)  $\sigma$  is steadily in libration but no Kozai resonance is found. (d)  $\sigma$  is steadily in libration and the perihelion argument,  $\omega$ , switches between libration and circulation. (e) Both  $\sigma$  and  $\omega$  are in libration.

Category (a) is the largest, which includes 14 plutinos, more than 40% of the total. The plutinos that would be expelled from the 3:2 resonance are listed in Table 2.

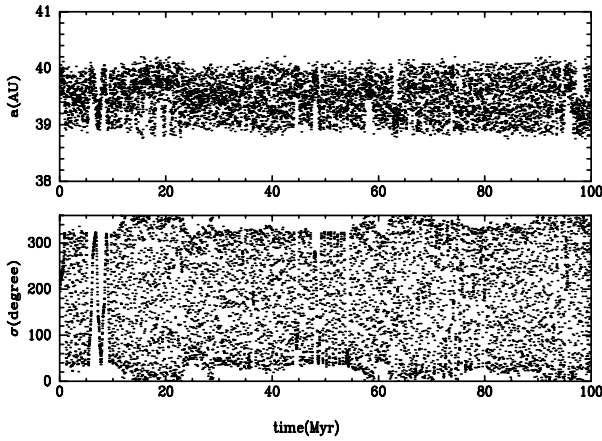
**Table 2.** Plutinos in category (a), which encounter Neptune or Pluto

Plutino	Time at encounter	
	with Neptune	with Pluto
1993SB	9.5 Myr	
1993SC	4.87 Myr	
1994JR1		81.78 Myr
1994TB	30 Kyr	
1995QZ9	26.76 Myr	
1996RR20		37.80 Myr
1996TP66	70 Kyr	
1997QJ4	29.72 Myr	
1998HH151		14 Kyr
1998HK151		3.15 Myr
1998HQ151		150 Kyr
1998UU43	120 Kyr	
1998WU31	18.85 Myr	
2000FE8*	30.12 Myr	

In order to find the time of a sudden change of the semi-major axis of a plutino when it encounters Neptune or Pluto, we shortened the stepsize of our numerical integration. Table 2 lists the time of encounter with Neptune (Col. 2) or Pluto (Col. 3). To find the influence of Pluto's perturbation on the stability of the orbits of plutinos, we removed Pluto from our model and integrated the system again. We found that four plutinos, 1993SC, 1995QZ9, 1998WU31 and 2000FE8, now became stable and got well inside the 3:2 resonance, but five others, 1994JR1, 1996RR20, 1998HH151, 1998HK151 and 1998HQ151, would now encounter Neptune instead of Pluto and be expelled from the 3:2 resonance. This fact tells us that Pluto's perturbation has to be taken into account when computing orbits of plutinos. We would like to mention that most of former works (for example, Morbidelli et al. 1995; Duncan et al. 1995) did not include Pluto in their model.

Morbidelli et al. (1995) integrated the orbits of 1993RO, 1993SB and 1993SC for 500 Myr and found that the orbits of the latter two are stable. The difference between their and our results in regard to these two bodies is mainly due to the improvement of the orbital elements in the mean time. The semi-major axis of 1993SB (Epoch 1994 Sep. 5.0) has been increased from 39.42 AU to 39.60 AU and that of 1993SC (Epoch 1995 Mar. 24.0) from 39.47 to 39.81 AU (see Table 1, Morbidelli et al. 1995). For using their model and initial condition and our program, we were able to find these two plutinos in regular orbits.

Category (b) consists of four members: 1996TQ66, 1998UR43, 1998US43 and 1999TR11. Their critical arguments in the 3:2 mean motion resonance switch between libration and circulation. Figure 1 shows the time variation of the semi-major axis  $a$  and the critical argument  $\sigma$  of 1998UR43. We can see that  $a$  is well kept between 39 and 40 AU and the maximum of  $\sigma$  is greater than  $300^\circ$ , a signal of its proximity to chaos. These four plutinos should



**Fig. 1.** Time variation of the semi-major axis  $a$  and the critical argument  $\sigma$  of 1998UR43

be regarded as being in the chaotic separatrix zone of the 3:2 resonance.

1993RO, 1995HM5, 1995YY3, 1998WV24, 1998WZ24, 1998WZ31, 1999JK132 and 1999KS16 fall into category (c). They remain in the 3:2 resonance during the whole of our integration and show no sign of the Kozai resonance. Special attention should be paid to 1993RO. Morbidelli et al. (1995) has shown that it is in strong chaos but Duncan et al. (1995) showed that it is stable for more than 1 Gyr with a set of improved orbit elements. In our integration 1993RO is also very stable and its  $\sigma$  librates between  $100^\circ$  and  $260^\circ$ . Its semi-major axis adopted by Morbidelli et al. (1995) is 39.696 AU at 1994 Sep. 5.0 and ours is 39.507 AU at the same epoch. We did a  $10^8$  years integration of 1993RO with their model and initial elements but with our symmetric integrator. We did not find any chaotic behavior either but its  $\sigma$  librated between  $70^\circ$  and  $300^\circ$ . We suspect that the difference was probably caused by their adopted integrator being less accurate. The maximum  $300^\circ$  could be a signal that it is near the separatrix and an accurate integration would be necessary to trace its true evolution.

Malhotra (1996, Fig. 6) pointed out that there would exist two kinds of stable librators in the 3:2 Neptune resonance, aphelion librators and perihelion librators, according to it being near its aphelion or perihelion when closest to Neptune. All the plutinos in our categories (c), (d) and (e) are found to be aphelion librators. This is because their eccentricities are not large enough (see Table 1) and a large eccentricity would cause instability when more major planets besides Neptune are taken into consideration. We suspect that there are no realistic plutinos of the perihelion liblator type.

Category (d) has only 2 members: 1998WV31 and 1999JC132. Their perihelion arguments,  $\omega$ , switch between libration and circulation but their critical arguments  $\sigma$  are in libration. Consequently, they are always in the 3:2 mean motion resonance but come in and out of the separatrix region of the Kozai resonance. Figure 2a shows the  $\omega$  of 1999JC132, which alternately librating around

$90^\circ$  and  $270^\circ$ . Figure 2b displays the orbit of 1998WV31 in the  $(e, \omega)$  plane. It clearly shows that there exist two stable libration points at  $\omega = 90^\circ, 270^\circ$  and  $e \sim 0.25$ , and two unstable libration points at  $\omega = 0^\circ, 180^\circ$ . 1998WV31 is obviously situated in a chaotic separatrix zone of the Kozai resonance. Since Fig. 2b is a projection of a six dimensional orbit on a plane, one should expect a certain amount of dispersion of the points, but our above description is probably correct by the large area of the dispersion.

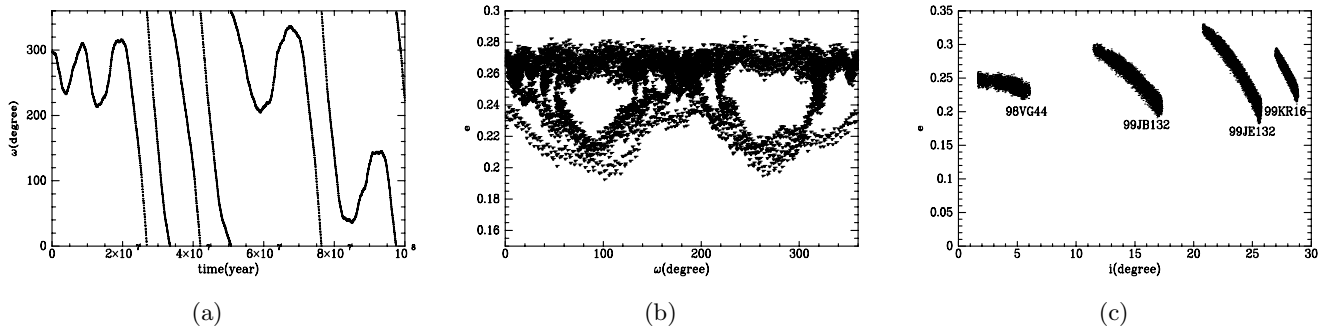
Category (e) contains objects inside the Kozai resonance. They are 4, namely, 1999JB132, 1999KR16, 1998WV24 and 1999JE132. The first two have their  $\omega$  librating around  $90^\circ$  and the last two around  $270^\circ$ . Compared to category (c), these four have smaller oscillation amplitude in semi-major axis, and larger oscillation amplitudes in eccentricity and inclination. Large oscillations in  $e$  and  $i$  excited by the Kozai resonance have been predicted by many authors, for example, Kozai (1962) and Thomas & Morbidelli (1996). A larger  $e$  would cause instability but a larger  $i$  accompanied with the Kozai resonance could increase stability. In the case of Pluto and plutinos, it is evident that the Kozai resonance increases their orbit stability and plays the role of protection mechanism by reducing the perturbation of Neptune. Another dynamical property (Thomas & Morbidelli 1996) that there should be a strong correlation between  $e$  and  $i$  for plutinos in the Kozai resonance can be clearly seen in Fig. 2c.

Pluto is in a third resonance, the so called 1:1 super resonance, as mentioned in the introduction section. We have not found any plutino in this resonance and believe that this protection mechanism is not important for orbital stability. Furthermore, we carefully checked all the possible secular resonances, perihelion to perihelion, node to node, but none of them were found.

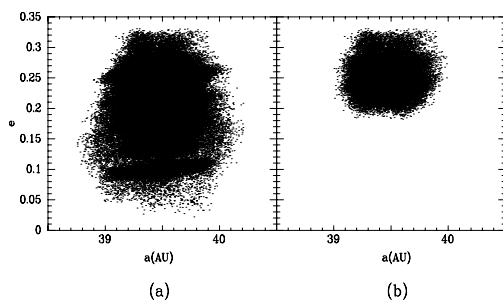
In order to find the stable resonance zone in the  $(a, e)$  plane and to compare our numerical result with the theoretical work of Morbidelli (1997, Fig. 8), we plot all the orbits of plutinos in our categories (b)-(e) in Fig. 3a, which outlines the 3:2 mean motion resonance zone. One can see that our distribution is smaller than Morbidelli's theoretical result from an averaged potential and in the limiting case of  $i = 0$ , but is quite consistent with Malhotra's numerical result from a planar restricted 3-body model (Malhotra et al. 2000, Fig. 3). Figure 3b shows the plot for categories (d) and (e) only. One can see that the stable Kozai resonance zone is inside the 3:2 resonance and occupies a small region around  $a = 39.5$  AU and  $e = 0.25$ . We did not find any example of Kozai resonance shown in the lower part (small eccentricity) of Fig. 8 of Morbidelli (1997).

#### 4. Discussion

It is hard to believe that so many plutinos (14 of 32, nearly 44%) are in unstable orbits (category (a)) after a long term evolution of the solar system. If this is true, the present KBOs do provide a rich source of short period comets. We suspect that the story will change when their orbits are



**Fig. 2.** a) Time variations of the perihelion argument  $\omega$  of 1999JC132. b) Projection of the orbit of 1998WV31 on the  $(e, \omega)$  plane. c) Correlation between  $e$  and  $i$  of the plutinos that are kept in the Kozai resonance



**Fig. 3.** a) Stable zone of the 3:2 mean motion resonance with Neptune from the orbit evolution of plutinos in categories b)–e). b) Stable zone of the Kozai resonance from the orbit evolution of plutinos in categories d) and e)

more precisely determined, although among the 14 members in category (a) only 2000FE8's orbit was determined from a single opposition. The orbits of 1993SB and 1993SC have been determined from seven oppositions but they are unstable from our computation.

Pluto's gravitation plays an important role on plutinos' orbital stability. In the numerical experiments of many previous authors, Pluto's perturbation was not included. Our computation shows that Pluto's perturbation would cause instability, as in the case of 1993SC, 1995QZ9, 1998WU31 and 2000FE8. Moreover, no 1:1 mean motion resonance with Pluto has been found for the 32 plutinos we studied.

Of the 18 stable plutinos (categories (b)–(e)) 6 (in categories (d) and (e)) are in the Kozai resonance or around its boundary. This is a quite large ratio. Further research on the Kozai resonance inside a mean motion resonance would be necessary. We were told that the behavior of the Kozai resonance depends on whether or not it is inside a mean motion resonance (Morbidelli 2000, private communication). A theoretical explanation of the behavior of 1998WV31 (Fig. 2b) should be worked out. The Kozai resonance zone of the six plutinos is confined to the upper part of the theoretical location worked out by Morbidelli (1997, Fig. 8). We do not understand why the Kozai resonance in nature happens only in a limited

region, much smaller than that predicted by Morbidelli's theory.

Pluto has three resonances to protect it from Neptune's perturbation. Up to now we have not found any plutino in the 1:1 super resonance. Why, then, is Pluto so peculiar? We now know that Pluto evolved into the 3:2 mean motion resonance by planets migration (Malhotra 1993). Malhotra (1995) also showed that Pluto and plutinos would be captured into the Kozai resonance with a certain probability due to planets migration. But Pluto's orbit is peculiar for the existence of the so called 1:1 super resonance and we know that the resonance zone of this super resonance is rather small (Wan et al. 2000). How did Pluto evolve into the 1:1 super resonance and why is it so different from the plutinos? This question should be addressed even though both the Kozai resonance and the 1:1 super resonance are not essential for Pluto's orbital stability (see Malhotra & Williams 1997).

Two important issues regarding our numerical results on the orbital stability of known plutinos must be recognized and emphasized. First, the orbits of these objects are not very well determined considering that their orbital periods are in excess of 250 years but the observational arcs are only 2–7 years long. Secondly, the phase space in the vicinity of the 3:2 Neptune resonance is known to be very small and to contain a complex mix of stable and unstable orbits. In other words, orbital stability in this region is a very sensitive function of initial conditions. Consequently, the error bars on the orbital elements of the observed plutinos may well span a range of stable and unstable orbits. Thus, it is not clear that the present 32 plutino orbits are a fair and representative sample of the real population. The presently estimated KBO orbits (even the multi-opposition ones) are still not “reliable”, and a need exists for an evaluation of the relationship between the sample of known orbits and the underlying real population.

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