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Regularities along spectral series in forbidden transitions of Ti^{11+}

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Abstract. Electric quadrupole (E2) transitions of Ti^{11+} have been computed with the Relativistic Quantum Defect Orbital (RQDO) method. The correctness of our predictions has been analysed through the inspection of the systematic behaviour of the RQDO line strengths along the spectral series, and by the excellent accord of the $S_{\rm RQDO}$ results with the scarce data available in the literature. Ti^{11+} being one of the ions of the iron group, the study of its forbidden lines is of particular interest. The spectrum of this ion is important for the study of plasmas in astrophysical objects and fusion devices.

Key words. atomic data

1. Introduction

The importance of forbidden lines in astrophysical and fusion studies has been pointed out by several authors (Biémont & Zeippen 1991). For those lines, astrophysicists and fusion specialists need accurate atomic data, such as radiative transition probabilities. To respond to this need, atomic physicists have put a great deal of effort into computing (and sometimes measuring) the required data. Particularly important is the study of the electric quadrupole (E2) forbidden lines, given that they are one of the bases for reliable electron density and/or temperature diagnosis in various astronomical objects or in laboratory tokamak plasmas (Biémont & Zeippen 1996).

The interest in line emission from highly-ionised atoms in tokamak devices is, in the first instance, due to the effect of impurities on the overall performance of the tokamak as a fusion device. The most common impurity elements are Ti, Cr, Fe, Ni, the lighter C, N, O, and the rare gases Ne and Ar (Peacock et al. 1984). In tokamaks, the forbidden lines are valuable diagnostic monitors of the ion motion (Stratton et al. 1983), and of the metal impurity concentrations (Bhatia & Mason 1980; Feldman et al. 1980). Even though the most prominent lines in Na-like ions correspond to electric-allowed transitions (Martín et al. 1991) the usefulness of E2 transition data in this context is widely recognised.

The determination of radiative transition probabilities (A), oscillator strengths (f) or line strengths (S) for the estimation of stellar chemical abundances is vital. It has been found that, at least for the abundant elements, the lines in the stellar spectra have rather small oscillator strengths or rather high excitation energies and are, thus, often difficult to measure experimentally. There are additional reasons for using absorption lines originating from rather highly excited states. Moreover, measurements of the trace metal abundance provide an opportunity to probe the chemical evolution of galaxies. An example is the determination of a Mn underabundance and a Ti overabundance which gives evidence of the nucleosynthetic signature of a gas cloud with a base metallicity similar to that of metal-poor galactic stars (Meyer et al. 1995).

The availability of intense tunable radiation sources has made it possible to measure the transition probabilities of some electric quadrupole (E2) transitions. Yet, an accurate assignment requires theoretical data. It is desirable, thus, to have a consistent set of accurate theoretical quadrupole Einstein coefficients for these forbidden lines. This, in turn, implies that accurate wavefunctions must be used for both initial and final states.

A number of ab initio codes are available, and some of them have proved to yield quite accurate transition probability data (Hibbert 1992). However, many of the adequate codes in this context are rather time-consuming and often plagued with convergence problems. It is widely recognised (Laughlin 1992, 1995) that for this type of study, the development of methods that combine reliability and simplicity is desiderable. In addition, some simple semiempirical techniques have revealed regularities that are not so well predicted by ab initio techniques. Furthermore, an appropiate semiempirical method may contain clues to an improved theoretical understanding of the dynamics of complex atoms (Curtis 1987). The differents types of regularities in atomic data have been amply studied (Wiese & Weiss 1968; Wiese 1987; Martin & Wiese 1996). The RQDO method has proved to reproduce the expected

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regularities in transition intensities along several isoelectronic sequences (see, e.g. Charro et al. 2000b) and homologous atomic systems (Martín et al. 2000a).

Ti^{11+} belongs to the sodium isolectronic sequence, with a [Ne]3s configuration for the ground state. Many calculations have been done for the ions isoelectronic with Na, most of them corresponding to allowed transitions. Several theoretical E2 S and f-values are in the literature. For the first three ions of the Na sequence, S-values have been reported (Tull et al. 1972a), and for the ions with $Z = 26$, 27 and 28, S-values also have been calculated by Tull et al. (1972b); along the sodium sequence up to $Z = 26$, oscillator strengths have been published by Biémont & Godefroid (1978); Godefroid et al. (1985) have reported values for $3\mathrm{s}^2\mathrm{S}-3\mathrm{d}^2\mathrm{D}$ forbidden transitions along the sodium sequence up to $Z = 26$. More recently, theoretical results about electric quadrupole transition probabilities for the Na-like ions Ba^{45+} through U^{81+} have been reported (Baik et al. 1991).

The most extensive data available in the literature for this ion are the multiplet gf-values calculated by Biémont & Godefroid (1978) using Hartree-Fock orbital wavefunctions. Transition probability data and S-values for forbidden lines of Ti^{11+} obtained with the same type of procedure are also reported in the critical compilation by Martin et al. (1988).

For many years we have applied the Quantum Defect Orbital method, both in its non-relativistic (QDO) (Simons 1974; Martín & Simons 1975, 1976) and its relativistic (RQDO) (Karwowski & Martín 1991; Karwowski & Martín 1991; Martín et al. 2000b) versions, to the calculation of oscillator strengths and photoionization cross sections of a rather large number of atomic species, of different degrees of complexity (Biémont et al. 1998; Charro et al. 1999, 2000a; Charro & Martín 1999a, 1999b, 2000a, 2000b), or including several isoelectronic sequences (see, e.g. Martín et al. 1991, 1993; Martín et al. 1994; Charro et al. 1996, 1997, 2000b; Charro & Martín 1998 and references therein).

The RQDO formalism is a simple but reliable analytical method based on a Hamiltonian model. It has the great advantage that the computational effort is not increased as the atomic system becomes heavier. It has been found that the RQDO orbitals behave rather well at intermediate and, in particular, large radial distances. These are, in most cases, the regions that contribute more strongly to the transition moment. The convenience of employing exactly solvable model potentials for calculating atomic transition probabilities manifests itself not only from a practical point of view but also because of the physical implications involved, when they are capable of achieving a good balance between computational effort and accuracy of results.

In the present work, electric quadrupole line strengths for fine-structure E2 transitions of Ti^{11+} have been computed with the RQDO formalism. The S-values have been calculated individually way, not from the application of the LS-coupling rules within multiplets. We find this interesting from a spectroscopic point of view, given their usefulness in spectral analysis in astrophysics and fusion plasma research. A general good agreement has been found between our data and other results.

2. Method of calculation

The Relativistic Quantum Defect Orbital (RQDO) method has been described in detail in previous papers (Karwowski & Martín 1991; Martín et al. 2000b). Therefore, we shall only briefly summarise its most fundamental aspects. The relativistic quantum defect orbitals are determined by solving analytically the quasi-relativistic, scalar, second-order Dirac-like equation, obtained after decoupling the radial, two-component Dirac equation, through a non-unitary transformation,

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{\Lambda(\Lambda+1)}{r^2} - \frac{2Z'_{\mathrm{net}}}{r}\right]\psi_k{}^{\mathrm{RD}} = 2e^{\mathrm{RD}}\psi_k{}^{\mathrm{RD}}, \qquad (1)$$

with

$$\Lambda = \eta - n + l - \delta' + c, \qquad (2)$$

$$Z'_{\mathrm{net}} = Z_{\mathrm{net}}(1+\alpha^2 E^x), \qquad (3)$$

$$e^{\mathrm{RD}} = -\frac{(Z'_{\mathrm{net}})^2}{2(\eta-\delta')^2} = E^x\frac{(1+\alpha^2E^x/2)}{(1+\alpha^2E^x)^2}\cdot \qquad (4)$$

η is the relativistic principal quantum number, n and l are the principal and orbital angular momentum quantum numbers, δ' is the relativistic quantum defect, c is an integer chosen to ensure the normalizability of the wavefunction and its correct nodal structure; Z'_{net} is the scaled nuclear charge acting on the valence electrons at large radial distances; E^x is the experimentally measured energy, and α is the fine structure constant. Atomic units are used throughout. Since the effective Hamiltonian in Eq. (2.1) includes a screening term, the quantum defect orbitals are approximately valid in the core region of space. Core polarization effects are implicitly included in the calculations as they are accounted for in the Λ parameter of the model Hamiltonian. On the other hand, given the one-electron nature of the RQDO formalism, it can be expected to perform better in highly excited states, where the active electron interacts less with the core and other valence electrons, than in low-lying energy states. The relativistic quantum defect orbitals lead to closed-form analytical expresions for the transition integrals. This allows us to avoid convergence problems in the calculation of transition probabilities and oscillator strengths.

Our methodology supplies one-electron radial wavefunctions, characterized by the n, l and j quantum numbers (Karwowski & Martín 1991), that we employ in the transition matrix elements for the initial and final states of the active electron. These correspond to levels of a given LSJ symmetry in many-electron atoms. We take care of the presence of the remaining electrons by including the appropriate angular factors in the line strengths. Thus, the electric quadrupole line strength for a transition between two states within the *LSJ*-coupling, which is the coupling scheme followed throughout in this work, is given by the equation

$$\begin{aligned} S_{\mathrm{E2}}(nlj, n'l'j') \;=\; & \frac{2}{3}\,(2J+1)\,(2J'+1)\,W(SJL'2, LJ')^2 \\ & \times R^2_{\mathrm{mult}}\,(\alpha L, \alpha' L')\,\langle L \parallel C^{(2)} \parallel L'\rangle^2 \\ & \times |\langle R_{nlj}|Q(r)|R_{n'l'j'}\rangle|^2 \end{aligned} \qquad (5)$$

where $Q(r)$ is the quadrupole transition operator, r^2, R_{mult} is the multiplet factor, $W(SJL'2, LJ')$ are the Racah coefficients which can be described in terms of 6j-symbols

$$W(SJL'2, LJ') = (-1)^{S+J+L'+2} \begin{Bmatrix} S & J & L \\ 2 & L' & J' \end{Bmatrix} \qquad (6)$$

and $\langle L \parallel C^{(2)} \parallel L'\rangle$ is the pertinent reduced matrix element, which can be evaluated using 3j-symbols by the following expresion

$$\langle L \parallel C^{(2)} \parallel L'\rangle = [L, L'] \begin{pmatrix} L & 2 & L' \\ 0 & 0 & 0 \end{pmatrix}. \qquad (7)$$

3. Numerical results

In the RQDO context, the quantum defects are extracted from energy level data. In this calculation we have employed the energies given in the critical compilation by Sugar & Corliss (1985). The measured energy levels have been reported to have an uncertainty estimated between 20 and 500 cm^{-1}. The value of the ionization energy, 2 351 080 cm^{-1}, was derived by Edlén (1978) from core polarization theory applied to the *n*f series. For those levels for which no data were found, such as the high *n*f states, we derived the corresponding energies making use of the quantum defect extracted from the energies of the highest *n*f levels available, that is 8f ($\delta'_{5/2} = 0.009803$ and $\delta'_{7/2} = 0.009480$), on the basis of Eq. (2.4), the grounds that the *n*f spectral serie of Ti^{11+} is not affected by configuration mixing.

The electric quadrupole gf-values for numerous multiplets in this sodiumlike ion determined by Biémont & Godefroid (1978) through a fully variational Hartree-Fock approach did not account for configuration mixing. The authors checked the accuracy of their wavefunctions using the appropiate relations of the hypervirial theorem. Their f-values were converted to multiplet strengths, and LS-coupling rules were applied to obtain strengths of lines within multiplets. Biémont & Godefroid (1978) reported values for all E2 transitions for the principal quantum number $n = 8$ for orbitals of s, p, d and f type.

The fine-structure S-values for the electric quadrupole transitions (E2), between doublets of Na-like Ti, the object of the present study, are displayed in Tables 1 and 2. For each transition for which comparative data were available, two sets of S-values are given, one obtained with the RQDO formalism, and the other including results from the critical compilation of Martin et al. (1988), derived from Biémont & Godefroid (1978) by using the *LS*-coupling rules. No further data relative to line strengths are, to our knowledge, available in the literature.

In Table 1, S-values for $n\mathrm{p}\,^2\mathrm{P} - n\mathrm{f}\,^2\mathrm{F}$ transitions with $n = 3{-}11$ and $n' = 4{-}11$ are collected. A general good accord between the set of RQDO S-values and those reported by Biémont & Godefroid (1978) is apparent. A maximum of 10% of relative deviation of the RQDO values is found here, with only the exception of the $3\mathrm{p}\,^2\mathrm{P}_{3/2} - 6\mathrm{f}\,^2\mathrm{F}_{7/2}$ transition, where the relative deviation is about 20%. It should not be overlooked, however, that the magnitude of the two compared quantities is very small. Results for $n\mathrm{f}\,^2\mathrm{F} - n\mathrm{p}\,^2\mathrm{P}$ transitions with n = 4–10 and n' = 5–11 are included in Table 2. Our RQDO data also conform satisfactorily with the comparative ones with a maximum deviation of 10%.

In order to apply another test to the quality of our results, we have plotted the RQDO S-values, multiplied by $(n'^{*})^3$, against (n'^{*}), the effective quantum number of the transition's upper state. It has long been established that for all spectral series of hydrogen or hydrogenlike species, the square of the radial integral, to which the line strength is proportional, diminishes as n'^{-3}, with n' being the principal quantum number (Martin & Wiese 1996). If the quantum defects are not small, as in atomic species other than hydrogenlike, n' should be replaced by the effective principal quantum number. The variation of the line strength with n'^{*} as we advance in an unperturbed spectral series, i.e., for sufficiently high n'^{*}, when the behaviour becomes hydrogenic, it satisfies (Martin & Wiese 1996):

$$S(nlj, n'l'j') \propto \; (n'^{*})^{-3}. \qquad (8)$$

In Figs. 1 to 5, the RQDO line strengths multiplied by the third power of the effective quantum number of the final state are plotted against n'^{*}, for several of the fine-structure transitions belonging to the spectral series object of the present work. It can be observed that the trend in our data is such that a constant value is reached as soon as the upper state is suficiently excited to acquire a near-hydrogen character. This is another proof of the correctness of the RQDO wavefunctions, given that the studied spectral series are free from perturbations by others.

4. Concluding remarks

The RQDO formalism has been followed to study E2 transitions in Na-like Ti, where the E2 line strengths of several spectral series of Ti^{11+} have been analysed. On the basis of the general good agreement between the RQDO and the comparative S-values, as well as the compliance of the present results with the above-mentioned feature of the LSJ coupling scheme, and with the expected systematic trends along different spectral series, we are confident of the usefulness of the data supplied in the present work for the fields of astrophysics and fusion plasma research. The RQDO procedure has once more proved to be a very

Table 1. Line strengths for the np → n'f fine-structure transitions.

Transition	Multiplet	J	J'	*RQDO*[a]	*Compilation*[b]
3p → 4f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.862	0.79
		3/2	5/2	0.250	0.225
		3/2	7/2	0.150(+1)	0.135(+1)
3p → 5f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.451(−1)	0.45(−1)
		3/2	5/2	0.127(−1)	0.13(−1)
		3/2	7/2	0.761(−1)	0.78(−1)
3p → 6f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.708(−2)	-
		3/2	5/2	0.194(−2)	-
		3/2	7/2	0.117(−1)	0.14(−1)
3p → 7f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.190(−2)	-
		3/2	5/2	0.508(−3)	-
		3/2	7/2	0.306(−2)	-
3p → 8f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.687(−3)	-
		3/2	5/2	0.180(−3)	-
		3/2	7/2	0.108(−2)	-
3p → 9f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.300(−3)	-
		3/2	5/2	0.768(−4)	-
		3/2	7/2	0.464(−3)	-
3p → 10f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.152(−3)	-
		3/2	5/2	0.380(−4)	-
		3/2	7/2	0.230(−3)	-
3p → 11f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.849(−4)	-
		3/2	5/2	0.209(−4)	-
		3/2	7/2	0.126(−3)	-
4p → 4f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.523(+1)	0.50(+1)
		3/2	5/2	0.150(+1)	0.143(+1)
		3/2	7/2	0.898(+1)	0.86(+1)
4p → 5f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.557(+1)	0.51(+1)
		3/2	5/2	0.163(+1)	0.146(+1)
		3/2	7/2	0.974(+1)	0.87(+1)
4p → 6f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.518	0.493
		3/2	5/2	0.148	0.141
		3/2	7/2	0.887	0.85
4p → 7f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.121	0.116
		3/2	5/2	0.340(−1)	0.33(−1)
		3/2	7/2	0.204	0.199
4p → 8f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.436(−1)	0.43(−1)
		3/2	5/2	0.122(−1)	0.12(−1)
		3/2	7/2	0.732(−1)	0.73(−1)
4p → 9f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.201(−1)	-
		3/2	5/2	0.559(−2)	-
		3/2	7/2	0.336(−1)	-
4p → 10f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.109(−1)	-
		3/2	5/2	0.302(−2)	-
		3/2	7/2	0.181(−1)	-
4p → 11f	$^{2}\mathrm{P}-{}^{2}\mathrm{F}$	1/2	5/2	0.658(−2)	-
		3/2	5/2	0.182(−2)	-
		3/2	7/2	0.109(−1)	-

Table 1. continued.

Transition	Multiplet	J	J'	$RQDO^a$	$Compilation^b$
5p → 5f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.497(+2)	0.477(+2)
		3/2	5/2	0.142(+2)	0.136(+2)
		3/2	7/2	0.854(+2)	0.82(+2)
5p → 6f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.229(+2)	0.216(+2)
		3/2	5/2	0.671(+1)	0.62(+1)
		3/2	7/2	0.402(+2)	0.370(+2)
5p → 7f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.249(+1)	0.234(+1)
		3/2	5/2	0.716	0.67
		3/2	7/2	0.429(+1)	0.402(+1)
5p → 8f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.638	0.60
		3/2	5/2	0.181	0.172
		3/2	7/2	0.109(+1)	0.103(+1)
5p → 9f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.245	-
		3/2	5/2	0.691(−1)	-
		3/2	7/2	0.415	-
5p → 10f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.118	-
		3/2	5/2	0.333(−1)	-
		3/2	7/2	0.200	-
5p → 11f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.662(−1)	-
		3/2	5/2	0.186(−1)	-
		3/2	7/2	0.112	-
6p → 6f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.261(+3)	0.250(+3)
		3/2	5/2	0.746(+2)	0.71(+2)
		3/2	7/2	0.448(+3)	0.430(+3)
6p → 7f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.731(+2)	0.69(+2)
		3/2	5/2	0.214(+2)	0.197(+2)
		3/2	7/2	0.129(+3)	0.118(+3)
6p → 8f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.846(+1)	0.80(+1)
		3/2	5/2	0.244(+1)	0.228(+1)
		3/2	7/2	0.146(+2)	0.137(+2)
6p → 9f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.224(+1)	-
		3/2	5/2	0.639	-
		3/2	7/2	0.384(+1)	-
6p → 10f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.876	-
		3/2	5/2	0.249	-
		3/2	7/2	0.150	-
6p → 11f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.430	-
		3/2	5/2	0.122	-
		3/2	7/2	0.731	-
7p → 7f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.998(+3)	0.970(+3)
		3/2	5/2	0.285(+3)	0.280(+3)
		3/2	7/2	0.171(+4)	0.1700(+4)
7p → 8f	$^2\mathrm{P}-{}^2\mathrm{F}$	1/2	5/2	0.206(+3)	0.192(+3)
		3/2	5/2	0.589(+2)	0.55(+2)
		3/2	7/2	0.353(+3)	0.330(+3)

Table 1. continued.

Transition	Multiplet	J	J'	$RQDO^a$	$Compilation^b$
7p → 9f	^{2}P – ^{2}F	1/2	5/2	0.238(+2)	-
		3/2	5/2	0.680(+1)	-
		3/2	7/2	0.408(+2)	-
7p → 10f	^{2}P – ^{2}F	1/2	5/2	0.630(+1)	-
		3/2	5/2	0.180(+1)	-
		3/2	7/2	0.108(+2)	-
7p → 11f	^{2}P – ^{2}F	1/2	5/2	0.246(+1)	-
		3/2	5/2	0.704	-
		3/2	7/2	0.422(+1)	-
8p → 8f	^{2}P – ^{2}F	1/2	5/2	0.309(+4)	0.2900(+4)
		3/2	5/2	0.884(+3)	0.830(+3)
		3/2	7/2	0.530(+4)	0.5000(+4)
8p → 9f	^{2}P – ^{2}F	1/2	5/2	0.494(+3)	-
		3/2	5/2	0.141(+3)	-
		3/2	7/2	0.846(+3)	-
8p → 10f	^{2}P – ^{2}F	1/2	5/2	0.572(+2)	-
		3/2	5/2	0.165(+2)	-
		3/2	7/2	0.987(+2)	-
8p → 11f	^{2}P – ^{2}F	1/2	5/2	0.153(+2)	-
		3/2	5/2	0.437(+1)	-
		3/2	7/2	0.262(+2)	-
9p → 9f	^{2}P – ^{2}F	1/2	5/2	0.824(+2)	-
		3/2	5/2	0.235(+4)	-
		3/2	7/2	0.141(+5)	-
9p → 10f	^{2}P – ^{2}F	1/2	5/2	0.105(+4)	-
		3/2	5/2	0.300(+3)	-
		3/2	7/2	0.180(+4)	-
9p → 11f	^{2}P – ^{2}F	1/2	5/2	0.124(+3)	-
		3/2	5/2	0.356(+2)	-
		3/2	7/2	0.213(+3)	-
10p → 10f	^{2}P – ^{2}F	1/2	5/2	0.196(+5)	-
		3/2	5/2	0.561(+4)	-
		3/2	7/2	0.336(+5)	-
10p → 11f	^{2}P – ^{2}F	1/2	5/2	0.208(+4)	-
		3/2	5/2	0.594(+3)	-
		3/2	7/2	0.356(+4)	-
11p → 11f	^{2}P – ^{2}F	1/2	5/2	0.431(+5)	-
		3/2	5/2	0.123(+5)	-
		3/2	7/2	0.738(+5)	-

a) Relativistic Quantum Defect Orbital Method, this work.

b) Martin et al. (1988).

In all the tables, $A(B)$ denotes $A \cdot 10^{(B)}$.

Table 2. Line strengths for the nf → n'p fine-structure transitions.

Transition	Multiplet	J	J'	*RQDO*[a]	*Compilation*[b]
4f → 5p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.105(+1)	0.113(+1)
		5/2	3/2	0.294	0.324
		7/2	3/2	0.177(+1)	0.194(+1)
4f → 6p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.351(−1)	0.38(−1)
		5/2	3/2	0.100(−1)	0.11(−1)
		7/2	3/2	0.603(−1)	0.65(−1)
4f → 7p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.728(−2)	-
		5/2	3/2	0.208(−2)	-
		7/2	3/2	0.125(−1)	0.13(−1)
4f → 8p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.267(−2)	-
		5/2	3/2	0.763(−3)	-
		7/2	3/2	0.458(−2)	-
4f → 9p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.128(−2)	-
		5/2	3/2	0.366(−3)	-
		7/2	3/2	0.220(−2)	-
4f → 10p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.721(−3)	-
		5/2	3/2	0.206(−3)	-
		7/2	3/2	0.124(−2)	-
4f → 11p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.459(−3)	-
		5/2	3/2	0.131(−3)	-
		7/2	3/2	0.786(−3)	-
5f → 6p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.113(+2)	0.119(+2)
		5/2	3/2	0.316(+1)	0.340(+1)
		7/2	3/2	0.190(+2)	0.204(+2)
5f → 7p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.337	0.353
		5/2	3/2	0.963(−1)	0.101
		7/2	3/2	0.578	0.61
5f → 8p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.654(−1)	0.68(−1)
		5/2	3/2	0.187(−1)	0.20(−1)
		7/2	3/2	0.112	0.117
5f → 9p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.228(−1)	-
		5/2	3/2	0.651(−2)	-
		7/2	3/2	0.391(−1)	-
5f → 10p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.106(−1)	-
		5/2	3/2	0.304(−2)	-
		7/2	3/2	0.182(−1)	-
5f → 11p	$^{2}\mathrm{F}-{}^{2}\mathrm{P}$	5/2	1/2	0.600(−2)	-
		5/2	3/2	0.172(−2)	-
		7/2	3/2	0.103(−1)	-

Table 2. continued.

Transition	Multiplet	J	J'	$RQDO^a$	$Compilation^b$
6f → 7p	^{2}F – ^{2}P	5/2	1/2	0.622(+2)	0.67(+2)
		5/2	3/2	0.178(+2)	0.190(+2)
		7/2	3/2	0.107(+3)	0.114(+3)
6f → 8p	^{2}F – ^{2}P	5/2	1/2	0.174(+1)	0.181(+1)
		5/2	3/2	0.498	0.52
		7/2	3/2	0.299(+1)	0.310(+1)
6f → 9p	^{2}F – ^{2}P	5/2	1/2	0.316	-
		5/2	3/2	0.904(−1)	-
		7/2	3/2	0.542	-
6f → 10p	^{2}F – ^{2}P	5/2	1/2	0.106	-
		5/2	3/2	0.304(−1)	-
		7/2	3/2	0.182	-
6f → 11p	^{2}F – ^{2}P	5/2	1/2	0.496(−1)	-
		5/2	3/2	0.142(−1)	-
		7/2	3/2	0.849(−1)	-
7f → 8p	^{2}F – ^{2}P	5/2	1/2	0.255(+3)	0.270(+3)
		5/2	3/2	0.728(+2)	0.77(+2)
		7/2	3/2	0.438(+3)	0.462(+3)
7f → 9p	^{2}F – ^{2}P	5/2	1/2	0.644(+1)	-
		5/2	3/2	0.184(+1)	-
		7/2	3/2	0.110(+2)	-
7f → 10p	^{2}F – ^{2}P	5/2	1/2	0.111(+1)	-
		5/2	3/2	0.316	-
		7/2	3/2	0.190(+1)	-
7f → 11p	^{2}F – ^{2}P	5/2	1/2	0.370	-
		5/2	3/2	0.106	-
		7/2	3/2	0.634	-
8f → 9p	^{2}F – ^{2}P	5/2	1/2	0.847(+3)	-
		5/2	3/2	0.242(+3)	-
		7/2	3/2	0.145(+4)	-
8f → 10p	^{2}F – ^{2}P	5/2	1/2	0.193(+2)	-
		5/2	3/2	0.551(+1)	-
		7/2	3/2	0.331(+2)	-
8f → 11p	^{2}F – ^{2}P	5/2	1/2	0.326(+1)	-
		5/2	3/2	0.931	-
		7/2	3/2	0.558(+1)	-
9f → 10p	^{2}F – ^{2}P	5/2	1/2	0.238(+4)	-
		5/2	3/2	0.680(+3)	-
		7/2	3/2	0.409(+4)	-
9f → 11p	^{2}F – ^{2}P	5/2	1/2	0.508(+2)	-
		5/2	3/2	0.145(+2)	-
		7/2	3/2	0.871(+2)	-
10f → 11p	^{2}F – ^{2}P	5/2	1/2	0.575(+4)	-
		5/2	3/2	0.164(+4)	-
		7/2	3/2	0.987(+4)	-

See footnotes in Table 1.

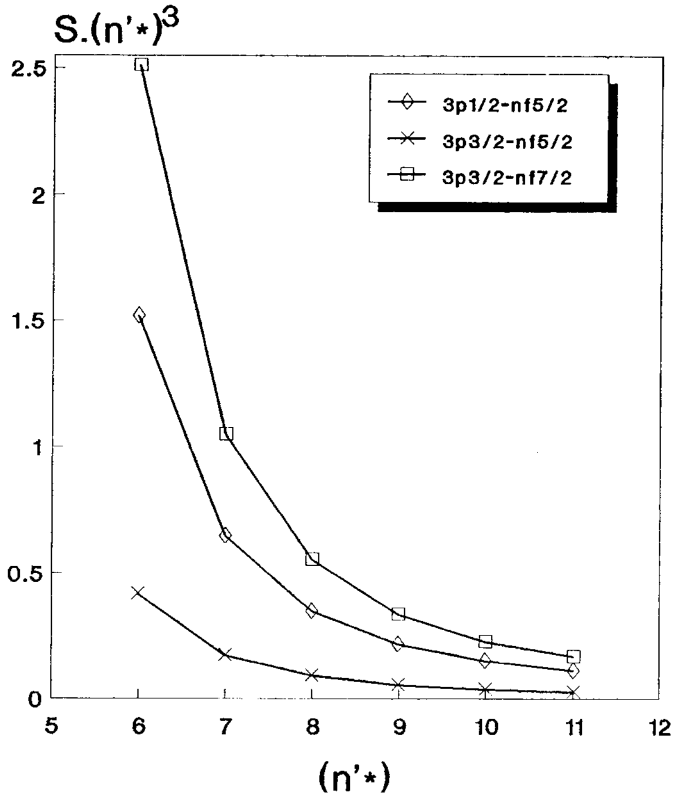

Fig. 1. Regularities in the line strength along the spectral series 3p^{2}P$_{1/2}$ − nf^{2}F$_{5/2}$, 3p^{2}P$_{3/2}$− nf^{2}F$_{5/2}$, and 3p^{2}P$_{3/2}$ − n f^{2}F$_{7/2}$.

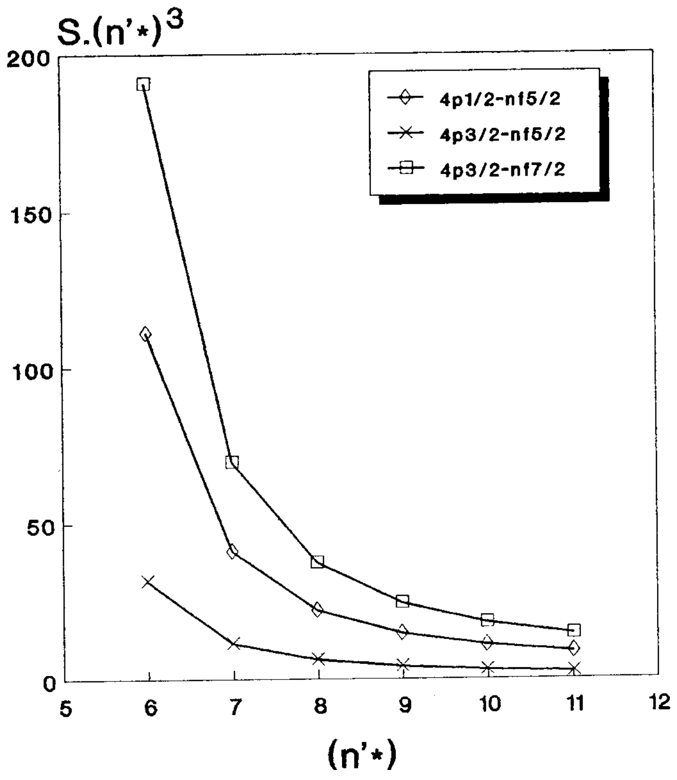

Fig. 2. Regularities in the line strength along the spectral series 4p^{2}P$_{1/2}$ − nf^{2}F$_{5/2}$, 4p^{2}P$_{3/2}$ − nf^{2}F$_{5/2}$, and 4p^{2}P$_{3/2}$ − nf^{2}F$_{7/2}$.

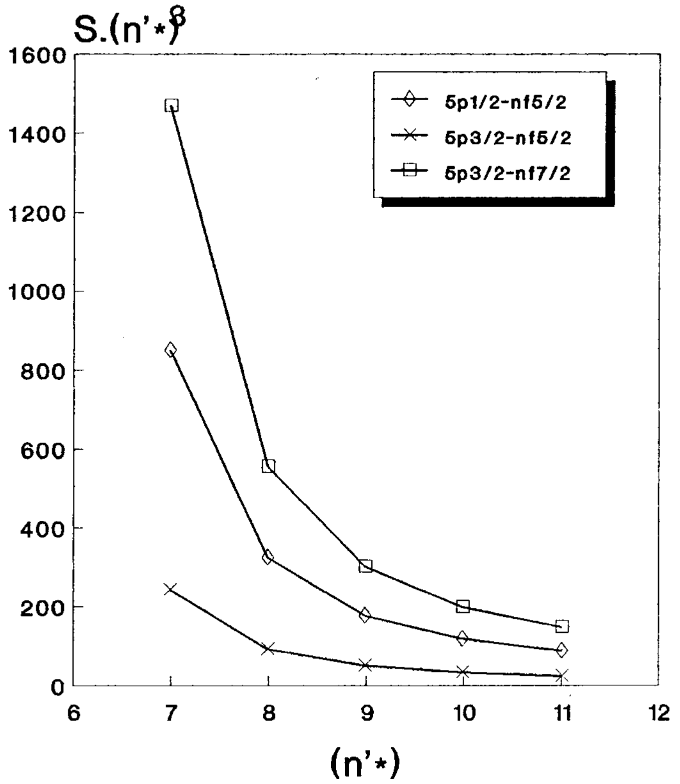

Fig. 3. Regularities in the line strength along the spectral series 5p ^{2}P$_{1/2}$− nf^{2}F$_{5/2}$, 5p ^{2}P$_{3/2}$ − nf^{2}F$_{5/2}$, and 5p ^{2}P$_{3/2}$− nf^{2}F$_{7/2}$.

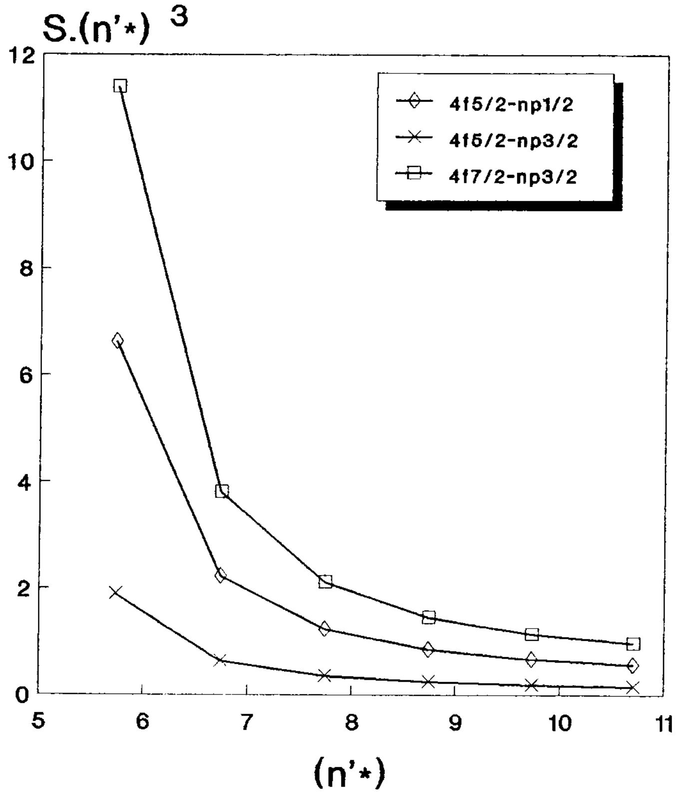

Fig. 4. Regularities in the line strength along the spectral series 4f^{2}F$_{5/2}$ − np^{2}P$_{1/2}$, 4f^{2}F$_{5/2}$ − np^{2}P$_{3/2}$ and 4f^{2}F$_{7/2}$− np^{2}P$_{3/2}$.

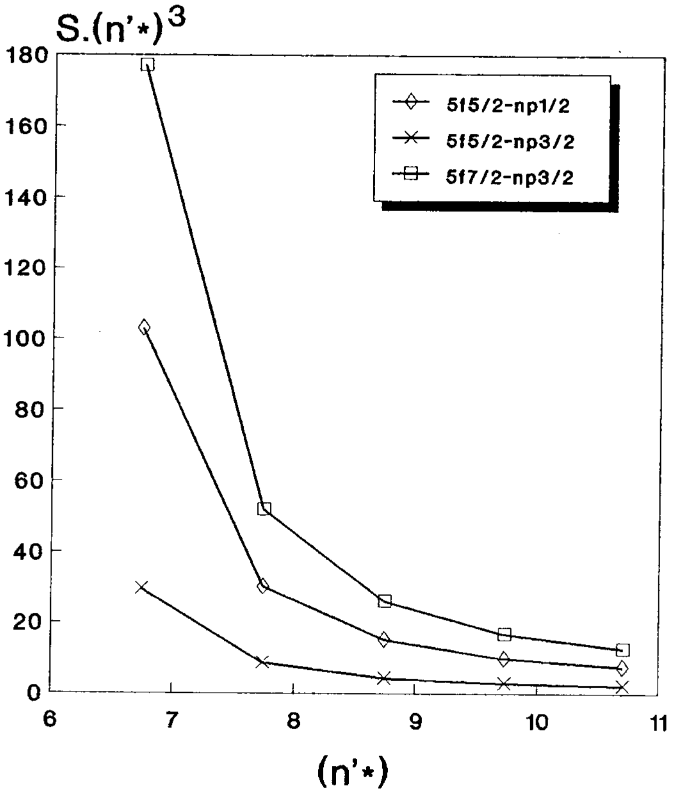

Fig. 5. Regularities in the line strength along the spectral series $5f^2F_{5/2}$ $-np^2P_{1/2}$, $5f^2F_{5/2}$ $-np^2P_{3/2}$ and $5f^2F_{7/2}-np^2P_{3/2}$.

useful tool for estimating transition intensities. It is particularly useful in the cases where mass-production of data is needed, given its cost-efficiency and reasonable accuracy. It offers also the advantages of being analytic and free of convergence problems.

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