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A natural explanation for periodic X-ray outbursts in Be/X-ray binaries

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Abstract. When applied to Be/X-ray binaries, the viscous decretion disc model, which can successfully account for most properties of Be stars, naturally predicts the truncation of the circumstellar disc. The distance at which the circumstellar disc is truncated depends mainly on the orbital parameters and the viscosity. In systems with low eccentricity, the disc is expected to be truncated at the 3:1 resonance radius, for which the gap between the disc outer radius and the critical lobe radius of the Be star is so wide that, under normal conditions, the neutron star cannot accrete enough gas at periastron passage to show periodic X-ray outbursts (type I outbursts). These systems will display only occasional giant X-ray outbursts (type II outbursts). On the other hand, in systems with high orbital eccentricity, the disc truncation occurs at a much higher resonance radius, which is very close to or slightly beyond the critical lobe radius at periastron unless the viscosity is very low. In these systems, disc truncation cannot be efficient, allowing the neutron star to capture gas from the disc at every periastron passage and display type I outbursts regularly. In contrast to the rather robust results for systems with low eccentricity and high eccentricity, the result for systems with moderate eccentricity depends on rather subtle details. Systems in which the disc is truncated in the vicinity of the critical lobe will regularly display type I outbursts, whereas those with the disc significantly smaller than the critical lobe will show only type II outbursts under normal conditions and temporary type I outbursts when the disc is strongly disturbed. In Be/X-ray binaries, material will be accreted via the first Lagrangian point with low velocities relative to the neutron star and carrying high angular momentum. This may result in the temporary formation of accretion discs during type I outbursts, something that seems to be confirmed by observations.

Key words. stars: circumstellar matter – emission-line, Be – binaries: close – neutron – X-ray: stars, bursts

1. Introduction

Be/X-ray binaries are X-ray sources composed of a Be star and a neutron star. The high-energy radiation is believed to arise owing to accretion of material associated with the Be star by the compact object (see Negueruela 1998; see also Bildsten et al. 1997).

A "Be star" is an early-type non-supergiant star, which at some time has shown emission in the Balmer series lines (Slettebak 1988, for a review). Both the emission lines and the characteristic strong infrared excess when compared to normal stars of the same spectral types are attributed to the presence of circumstellar material in a disc-like geometry. The causes that give rise to the disc are not well understood. Different mechanisms (fast rotation, non-radial pulsation, magnetic loops) have been proposed, but it is still unclear whether any of them can explain the

observed phenomenology on its own. The discs are rotationally dominated and motion seems to be quasi-Keplerian (Hanuschik 1996). However, some kind of global outflow is needed to explain the X-ray emission from Be/X-ray binaries (Waters et al. 1988).

Some Be/X-ray binaries are persistent X-ray sources (see Reig & Roche 1999), displaying low luminosity ($L_x \sim 10^{34}$ erg s^{-1}) at a relatively constant level (varying by up to a factor of $\sim$10). On the other hand, most known Be/X-ray binaries (though this is probably a selection effect) undergo periods in which the X-ray luminosity suddenly increases by a factor $\gtrsim$10 and are termed Be/X-ray transients.

Be/X-ray transients fall within a relatively narrow area in the $P_{\rm orb}/P_{\rm spin}$ diagram (see Corbet 1986; Waters & van Kerkwijk 1989), indicating that some mechanism must be responsible for the correlation. Those systems with fast-spinning neutron stars do not show pulsed X-ray emission during quiescence (though non-pulsed radiation could be

caused by accretion on to the magnetosphere) because of the centrifugal inhibition of accretion (Stella et al. 1986). Systems with more slowly rotating pulsars show X-ray emission at a level $L_{\mathrm{x}} \lesssim 10^{35}$ erg s^{-1} when in quiescence. Transients show two different kinds of outbursts:

- X-ray outbursts of moderate intensity ($L_{\mathrm{x}} \approx 10^{36}$–$10^{37}$ erg s^{-1}) occurring in series separated by the orbital period (type I or normal), generally (but not always) close to the time of periastron passage of the neutron star. In most cases, the duration of these outbursts seems to be related to the orbital period;
- Giant (or type II) X-ray outbursts ($L_{\mathrm{x}} \gtrsim 10^{37}$ erg s^{-1}) lasting for several weeks or even months. Generally type II outbursts start shortly after periastron passage, but do not show any other correlation with orbital parameters (Finger & Prince 1997). In systems like 4U 0115+63 the duration of the type II outbursts seems to be to some degree correlated with its peak intensity, but in A 0535+262 type I outbursts may be as long as much brighter type II outbursts (Finger et al. 1996a).

2. Radial outflows vs. quasi-Keplerian discs

Attempts at modelling the X-ray luminosities of Be/X-ray transients during outbursts have made use of a simple wind accretion model, in which the neutron star accretes from a relatively fast radial outflow. The disc of the central Be star is supposed to have a power-law density distribution

$$\rho(r) = \rho_0 \left(\frac{r}{R_*} \right)^{-n} \tag{1}$$

where ρ_0 is the density at the stellar surface and R_* is the radius of the star. This results in a power velocity law of the form

$$v(r) = v_0 \left(\frac{r}{R_*} \right)^{n-2} \tag{2}$$

where the values of v_0 and n have to be determined observationally (see Waters et al. 1989 and references therein). The rotational velocity of the outflow takes the form

$$v_{\mathrm{rot}}(r) = v_{\mathrm{rot},0} \left(\frac{r}{R_*} \right)^{-\alpha} \tag{3}$$

with $0.5 \leq \alpha \leq 1$ (respectively the Keplerian case and conservation of angular momentum).

Accretion is considered to follow the classical Bondi-Hoyle-Littleton (BHL) approximation. The most important parameter, the relative velocity between the outflow and the neutron star, can be written as

$$v_{\mathrm{rel}}^2 = (v - V_{\mathrm{rad}})^2 + (v_{\mathrm{rot}} - V_{\mathrm{rot}})^2 \tag{4}$$

where V_{rad} and V_{rot} are the radial and tangential components of the orbital velocity of the neutron star. Material is supposed to be accreted when it is within a capture radius defined as

$$r_{\mathrm{c}} = 2GM_{\mathrm{x}}v_{\mathrm{rel}}^{-2} \tag{5}$$

where M_{x} is the mass of the neutron star and G is the gravitational constant. The X-ray luminosity in the BHL approximation can then be expressed as

$$L_{\mathrm{x}} = 4\pi G^3 M_{\mathrm{x}}^3 R_{\mathrm{x}}^{-1} v_{\mathrm{rel}}^{-4} F_{\mathrm{m}} \propto \rho v_{\mathrm{rel}}^{-3} \tag{6}$$

where R_{x} is the radius of the neutron star and $F_{\mathrm{m}} = \rho v_{\mathrm{rel}}$ is the mass flow. In order to explain the wide range of observed X-ray luminosities, large changes in the value of the radial velocity have to be invoked. For example, Waters et al. (1989) deduced that the relative velocity was $v_{\mathrm{rel}} \approx 300$ km s^{-1} during a type I outburst of V 0332+53 in 1983, while it was $\ll 100$ km s^{-1} during a type II outburst in 1973.

The use of the BHL approximation implies a number of simplifying assumptions which are not always easy to justify. For example, it neglects any effect of the mass-losing star, which for periastron distances of $\sim 10 R_*$ and mass ratios $q \sim 0.1$ seems to be an excessive simplification. In addition, while the use of the accretion radius formalism is adequate for an accreting object immersed in a medium, its application to an outflow characterised by a relatively small scale-height (H), such as a Be disc, is dubious.

Moreover, when the low outflow velocities required to explain type II outbursts are considered, the formalism breaks down completely, since the capture radius becomes far too large to have any physical meaning. For example, for $v_{\mathrm{rel}} \simeq 20$ km s^{-1}, $r_{\mathrm{c}} \simeq 9.6 \times 10^{11}$ m $\approx 1400 R_{\odot}$, which is one order of magnitude larger than the binary separation. A crude way round this problem is to consider that the radius of the effective Roche lobe of the neutron star r_{R} should be used instead of r_{c} whenever the calculated value is larger than r_{R} (e.g., Ikhnasov 2001). In spite of these shortcomings, the model has been repeatedly used in an attempt to model lightcurves of Be/X-ray binaries (Raguzova & Lipunov 1998; Reig et al. 1998) with only moderate success.

Beyond the purely formal aspects, one obvious difficulty for the model is the fact that many Be/X-ray binaries (e.g., A 0535+262) show low-luminosity X-ray emission when they are not in outburst. It is believed that all Be/X-ray binaries for which centrifugal inhibition of accretion is not effective display this emission. Motch et al. (1991) detected A 0535+262 on several occasions at luminosities of $\approx 2 \times 10^{35}$ erg s^{-1}. In order to explain this luminosity within the framework described above and taking into account that optical and infrared observations do not show any sign of the large variations that would be associated with a change of several orders of magnitude in the density of material, enormous relative velocities (of the order of $\sim 10^4$ km s^{-1}) are needed.

One further complication comes from the fact that Be/X-ray binaries spend most of their time in the quiescent state described in the previous paragraph and only

occasionally show series of outbursts. The model does not offer any explanation as to why there could be a change from quiescence to outburst, unless again very large and sudden changes in the density and velocity of the flow are assumed. Given the changes in relative velocities needed to account for the observed range of X-ray luminosities and the lack of any physical mechanism that could explain them, it is clear that direct accretion from a wind-like outflow is not the best approximation to the way in which matter is fed on to the neutron star.

But the major objection to the model is simply the fact that there is no observational evidence whatsoever supporting the existence of such fast outflows. All observations of Be stars imply bulk outflow velocities smaller than a few km s^{-1} (Hanuschik 2000). The evidence for rotationally dominated quasi-Keplerian discs around Be stars is overwhelming (see Hanuschik et al. 1996; Hummel & Hanuschik 1997; Okazaki 1997), especially owing to the success of the one-armed global oscillation model to explain V/R variability in the emission lines of Be stars (Kato 1983; Okazaki 1991, 1996; Papaloizou et al. 1992; Hummel & Hanuschik 1997).

Therefore it seems necessary to attempt an explanation of the outburst behaviour of Be/X-ray binaries that does not imply large outflow velocities.

3. The viscous disc model

Whatever the mechanism originating the Be phenomenon, the model which at present appears more applicable to explaining the discs surrounding Be stars is the viscous decretion disc model (Lee et al. 1991; see Porter 1999; Okazaki 2001 for detailed discussion). In this scenario, angular momentum is transferred from the central star by some mechanism still to be determined (perhaps associated with non-radial pulsations) to the inner edge of the disc, increasing its angular velocity to Keplerian. Viscosity then, operating in a way opposite to an accretion disc, conducts material outwards. In this scenario, material in the disc moves in quasi-Keplerian orbits and the radial velocity component is highly subsonic until the material reaches a distance much larger than the line-emitting regions (Okazaki 2001). The outflow is very subsonic for the distances at which neutron stars orbit in close Be/X-ray transients $\left(v(r) < 1 \text{ km s}^{-1}\right)$ and still subsonic for the orbital sizes of all Be/X-ray binaries for which there is an orbital solution. The viscous decretion disc model successfully accounts for most of the observational characteristics of Be discs.

Negueruela & Okazaki (2001, henceforth Paper I) have modelled the disc surrounding the Be primary in the Be/X-ray transient 4U 0115+63 as a viscous decretion disc and found that the tidal interaction of the neutron star naturally produces the truncation of the circumstellar disc, as it does for accretion discs in close binaries (Paczyński 1977). The result of Negueruela & Okazaki (2001) is in agreement with the results of Reig et al. (1997), who showed that there is a correlation between the orbital size and the maximum equivalent width of Hα ever observed in a system. Even though it is clear that the equivalent width of Hα is not an effective measurement of the size of the disc owing to several effects (see, for example, Negueruela et al. 1998), the maximum equivalent width ever observed becomes a significant indicator if the system has been monitored during a period which is long in comparison with the typical time-scale for changes in the disc (which, if viscosity is dominant, should be only a few months). Therefore the result of Reig et al. (1997) clearly indicates that the neutron star has some sort of effect on the size of the disc.

In this paper we apply the model presented in Paper I to several Be/X-ray transients for which orbital solutions exist and investigate how the truncation radius depends on different orbital parameters.

4. Model description and limitations

The model developed in Paper I describes a binary system in which a primary Be star of mass M_* and radius R_* is orbited by a neutron star of mass $M_x = 1.4\,M_\odot$ which moves in an orbit of eccentricity e and period $P_{\rm orb}$. The Be star is assumed to be surrounded by a near-Keplerian disc which is primarily governed by pressure and viscosity. For simplicity, the disc is assumed to be isothermal and Shakura-Sunyaev's viscosity prescription is adopted.

In such a disc, angular momentum is added to the disc by the viscous torque, whereas it is removed from the disc by the resonant torque exerted by the neutron star companion, which becomes non-zero only at radii where the ratio between the angular frequency of disc rotation and the angular frequency of the mean binary motion is a rational number. As a result, the disc decretes outward owing to the transfer of angular momentum by viscosity until the resonant torque becomes larger than the viscous torque at a resonant radius.

Therefore, the criterion for the disc truncation at a given resonance radius is written as

$$T_{\rm vis} + T_{\rm res} \leq 0, \tag{7}$$

where $T_{\rm vis}$ and $T_{\rm res}$ are the viscous torque and the resonant torque, respectively.

The viscous torque $T_{\rm vis}$ is written as

$$T_{\rm vis} = 3\pi\alpha G M_* \sigma r \left(\frac{H}{r}\right)^2 \tag{8}$$

(Lin & Papaloizou 1986), where σ is the surface density of the disc, α is the Shakura-Sunyaev viscosity parameter and H is the vertical scale-height of the disc given by

$$\frac{H}{r} = \frac{c_{\rm s}}{V_{\rm K}(R_*)} \left(\frac{r}{R_*}\right)^{1/2} \tag{9}$$

for the isothermal disc. Here, $c_{\rm s}$ is the isothermal sound speed and $V_{\rm K}(R_*)$ is the Keplerian velocity at the stellar surface. In the systems we will discuss later, $c_{\rm s}/V_{\rm K}(R_*)$ ranges 3.1–$4.1\times10^{-2}(T_{\rm d}/T_{\rm eff})^{1/2}$, where $T_{\rm d}$ and $T_{\rm eff}$ are

the disc temperature and the effective temperature of the Be star, respectively. Note that the viscous torque is proportional to the disc temperature.

The resonant torque T_{res} is calculated by using Goldreich & Tremaine's (1979, 1980) torque formula, after decomposing the binary potential Φ into a double Fourier series as

$$
\begin{aligned}
\Phi(r,\theta,z) &= -\frac{GM_*}{r} - \frac{GM_{\mathrm{x}}}{[r^2 + r_2^2 - 2rr_2\cos(\theta - f)]^{1/2}} \\
&\quad + \frac{GM_{\mathrm{x}}r}{r_2^2}\cos(\theta - f) \\
&= \sum_{m,l}\phi_{ml}\exp[i(m\theta - l\Omega_{\mathrm{B}}t)],
\end{aligned}
\tag{10}
$$

where r_2 is the distance of the neutron star from the primary, f is the true anomaly of the neutron star, m and l are the azimuthal and time-harmonic numbers, respectively, and $\Omega_{\mathrm{B}} = [G(M_* + M_{\mathrm{x}})/a^3]^{1/2}$ is the mean motion of the binary with semimajor axis a. The pattern speed of each potential component is given by $\Omega_{\mathrm{p}} = (l/m)\Omega_{\mathrm{B}}$. The third term in the right hand side of the first equation is the indirect potential arising because the coordinate origin is at the primary.

For each potential component, there can be three kinds of resonances, i.e., the outer and inner Lindblad resonances at radii $[(m \pm 1)/l]^{2/3}(1 + q)^{-1/3}a$, where $\Omega_{\mathrm{p}} = \Omega \pm \kappa/m$, and a corotation resonance (CR) at the radius $(m/l)^{2/3}(1 + q)^{-1/3}a$, where $\Omega_{\mathrm{p}} = \Omega$. Here, κ is the epicyclic frequency and the upper and lower signs correspond to the outer Lindblad resonance (OLR) and inner Lindblad resonance (ILR), respectively. In circumstellar discs, however, the resonant torque from the inner Lindblad resonance, which is given by

$$
(T_{ml})_{\mathrm{ILR}} = -\frac{m(m-1)\pi^2\sigma(\lambda + 2m)^2\phi_{ml}^2}{3l^2\Omega_{\mathrm{B}}^2},
\tag{11}
$$

where $\lambda = \mathrm{d}\ln\phi_{ml}/\mathrm{d}\ln r$, always dominates the resonant torques from the corotation resonance and the outer Lindblad resonance. The resonant torque at a given resonance radius is then given by

$$
\begin{aligned}
T_{\mathrm{res}} &= \sum_{ml}(T_{ml})_{\mathrm{ILR}} + \sum_{m'l'}(T_{m'l'})_{\mathrm{OLR}} + \sum_{m''l''}(T_{m''l''})_{\mathrm{CR}} \\
&\simeq \sum_{ml}(T_{ml})_{\mathrm{ILR}}.
\end{aligned}
\tag{12}
$$

Since high-order potential components contribute little to the total torque, the summation in Eq. (12) is safely taken over several lowest-order potential components which give the same radius.

For a given set of stellar and orbital parameters and disc temperature, criterion (7) at a given resonance is met for α smaller than a critical value α_{crit} and we assume that the disc is truncated at the resonance if $\alpha < \alpha_{\mathrm{crit}}$.

In Paper I it was shown that for any subsonic outflow, the drift time-scale $\tau_{\mathrm{drift}} \sim \Delta r/v_{\mathrm{r}} \sim \mathcal{M}_{\mathrm{r}}^{-1}(\Delta r/H)\Omega^{-1}$ was considerably longer than a typical truncation time-scale $\tau_{\mathrm{trunc}} \sim (\alpha/\alpha_{\mathrm{crit}})\tau_{\mathrm{vis}} \sim \alpha_{\mathrm{crit}}^{-1}(\Delta r/H)^2\Omega^{-1}$, where Δr

is the gap size between the truncation radius and the radius where the gravity by the neutron star begins to dominate, and v_{r} and $\mathcal{M}_{\mathrm{r}}$ are the radial velocity and Mach number, respectively. As a consequence, the tidal and resonant interaction with the neutron star leads to disc truncation.

One important consequence of the above is that the discs surrounding the primaries in Be/X-ray binaries cannot reach a steady state. Most of the outflowing material loses angular momentum and falls back towards the central star. As a consequence of the interaction with the material which is coming outwards from the inner regions, it is likely that the disc becomes denser and the density distribution in the radial direction becomes flatter with increasing time. We note that we do not expect the truncation effect to be one hundred per cent efficient. This is not only owing to theoretical considerations (see below), but probably required by the existence of pulsed low-luminosity X-ray emission during quiescence.

In the formulation above only torques integrated over the whole orbit are considered. Given the large eccentricities observed in Be/X-ray binaries (mostly larger than 0.3 and sometimes approaching 0.9), the gravitational effect of the neutron star is very strongly dependent on the orbital phase. This means that in such systems the disc radius and the truncation radius are also phase-dependent. The disc will shrink at periastron, at which the truncation radius becomes smallest, while it will spread when the neutron star is far away and its gravitational effect is not felt so strongly. The spread of the disc will continue until the truncation radius becomes smaller than the disc radius at some phase before the next periastron passage. This variation in disc radius will be larger for a larger eccentricity. For a longer orbital period, the effect will be yet stronger. Given that, for some systems considered (see Sect. 5), the model truncation radius is close to the critical lobe radius at periastron, this will provide a mechanism by which disc material can reach the neutron star (see also Fig. 3 for scenarios for two families of type I outbursts).

5. Modelling the Be/X-ray binaries

Table 1 shows a list of known X-ray binaries which have exhibited outbursting behaviour or some sort of orbital modulation in their X-ray lightcurve. The top panel contains Be/X-ray binaries with an identified Be counterpart which have displayed type I X-ray outbursts, i.e., a series of outbursts separated by their orbital period (note that in many cases there is no orbital solution for the system and the recurrence period of the outbursts is taken to be the orbital period). The middle panel shows other systems without identified optical counterparts whose X-ray behaviour marks them out as Be/X-ray binaries. Finally the bottom panel contains a few other Be/X-ray binaries with identified counterparts whose X-ray behaviour deviates slightly from what is considered typical (and which will be discussed individually in Sect. 6).

In this section, we apply our model to systems for which exact orbital solutions have been deduced from the analysis of Doppler shifts in the arrival times of X-ray photons. They include five Be/X-ray binaries in the top panel (V 0332+53, A 0535+262, GRO J1008−57, 2S 1417−624, and EXO 2030+375) and a likely Be/X-ray binary in the middle panel (2S 1845−024). These systems are discussed individually in the following subsections. The case of the Be/X-ray transient 4U 0115+63 has been carefully discussed in Paper I and therefore it will not be included here.

As mentioned in the previous section, we adopt Shakura-Sunyaev's viscosity prescription, in which the viscosity parameter α is a free parameter, and assume the Be disc to be isothermal. In what follows, we adopt $T_d = \frac{1}{2}T_{\rm eff}$. Note that our assumption of the disc temperature is consistent with the results by Millar & Marlborough (1998, 1999), who computed the distribution of the disc temperature within $100\,R_*$ around the B0 star γ Cas and the B8−9 star 1 Del by balancing at each position the rates of energy gain and energy loss and found that the disc is roughly isothermal at a temperature about half the effective temperature of the star.

When adopting a particular model for a system, the main source of uncertainty comes from the choice of mass for the primary, a parameter which can only be guessed from the spectral type. We find two main difficulties. In some cases, the spectral type of the primary is not well determined. Moreover, there is some evidence that fast rotators may be moderately over-luminous for their masses (see Gies et al. 1998). For this reason, in general we have taken masses slightly lower than those given in the calibration of Vacca et al. (1996). The spectral distribution of primaries of Be/X-ray binaries (Negueruela 1998) is strongly peaked at B0. Therefore for systems without exact determination of the spectral type, we have calculated models corresponding to B0V and B0III primaries. In any case, our results show that the exact mass of the primary is not one of the main factors in the X-ray behaviour of the sources.

5.1. V 0332+53

This transient pulsar has an orbital period $P_{\rm orb} = 34.25$ d with a relatively low eccentricity of $e = 0.31$ (Stella et al. 1985). The optical component of this system is an unevolved star in the O8−9 range (Negueruela et al. 1999). For O8.5V stars Vacca et al. (1996) give an spectroscopic mass $M_* = 23.6\,M_\odot$ and a theoretical mass $M_* = 28\,M_\odot$. Here we will assume as a conservative model the lower limit of $M_* = 20\,M_\odot$ and $R_* = 8.8\,R_\odot$. The mass function $f(M) = 0.1$ then implies $\sin i = 0.17$, resulting in an orbital separation $a \simeq 130\,R_\odot$ ($r_{\rm per} \approx 10\,R_*$).

This system has been rarely detected in X-rays. A type II outburst was observed in 1973 (see Negueruela et al. 1999 for references). Ten years later, it was observed during a series of three type I outbursts. Finally
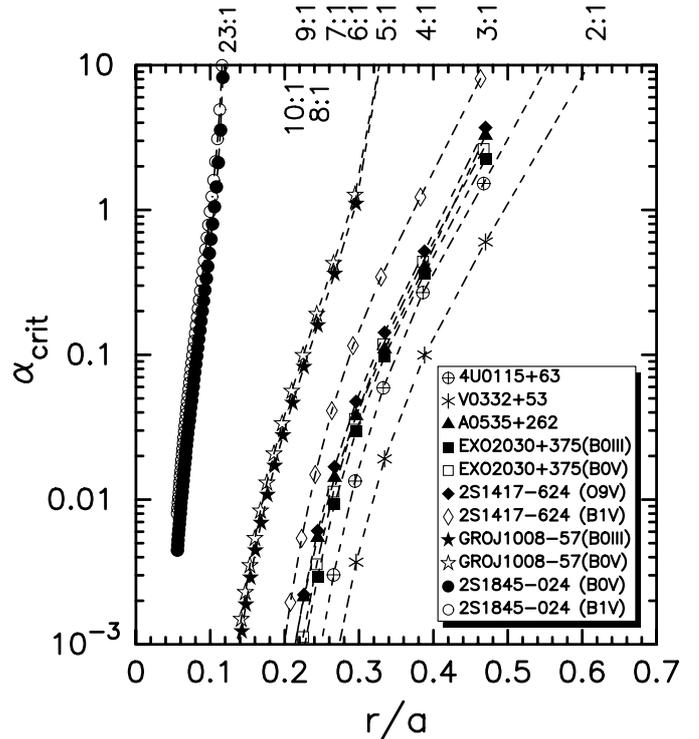

Fig. 1. Critical values of α at some resonance radii for systems discussed in the text. Annotated in the figure are the locations of the $n{:}1$ commensurabilities of disc and mean binary orbital frequencies. $T_d = \frac{1}{2}T_{\rm eff}$ is adopted for all models. For other disc temperatures, $\alpha_{\rm crit}$ should be multiplied by a factor of $T_{\rm eff}/2T_d$.

it was observed during another type II outburst in 1989. Between 1991 and 2000 it has not been detected by either the BATSE instrument on board the *Compton GRO* satellite or the All Sky Monitor on board *RossiXTE* and it is believed to be in a dormant state (Negueruela et al. 1999).

In Fig. 1, we plot $\alpha_{\rm crit}$ at the $n{:}1$ resonance radii for those Be/X-ray binaries which will be discussed in this section. The resonant torques at the $n{:}1$ radii are stronger than those at radii with other period commensurabilities located nearby. Note that we have adopted a particular disc temperature, $T_d = \frac{1}{2}T_{\rm eff}$, for each stellar model. For other disc temperatures, $\alpha_{\rm crit}$ should be multiplied by a factor of $T_{\rm eff}/2T_d$, taking account of the fact that the viscous torque is proportional to T_d.

Figure 1 shows that the Be disc in V 0332+53 is expected to be truncated at the 3:1 resonance radius ($r_t/a \simeq 0.47$, where r_t is the truncation radius) for $0.099 \lesssim \alpha \lesssim 0.60$ and at the 4:1 resonance radius ($r_t/a \simeq 0.39$) for $0.019 \lesssim \alpha \lesssim 0.099$.

It is interesting to see how close the truncation radius is to the size of the critical lobe at periastron. Figure 2 shows orbital models for the systems discussed in this section. The potential ψ describing the effects of the gravitational and centrifugal forces on the motion of test particles orbiting the Be star is given by

$$\psi(r, \theta, z) = \Phi(r, \theta, z) - \frac{1}{2}\Omega^2(r)r^2, \qquad (13)$$

Table 1. List of known X-ray binaries to which our model could be applied. The top panel contains Be/X-ray binaries with an identified counterpart which have displayed type I X-ray outbursts. The middle panel shows other systems without identified optical counterparts likely to be Be/X-ray binaries. The bottom panel contains some other Be/X-ray binaries with identified counterparts in which the nature of the outbursts is not clear yet (all these systems are discussed in the text). Basic orbital parameters are listed. Orbital periods marked with "*" represent the recurrence time of X-ray outbursts; other orbital periods are derived from exact orbital solutions. Spectral types have only been included for those systems in which they are derived from high signal-to-noise ratio spectra in the classification region. Objects in which the eccentricity is marked as "large" have no orbital solutions, but available data imply $e > 0.5$. References are only given if data were not included in Negueruela (1998).

Name	Optical Counterpart	Spectral Type	P_s(s)	P_{orb}(d)	e
4U 0115+634	V635 Cas	B0.2V[a]	3.6	24.3	0.34
V 0332+53	BQ Cam	O8.5V[b]	4.4	34.2	0.31
A 0535+262	V725 Tau	B0III	103	111	0.47
RX J0812.4−3114	LS 992	B0.2III[c]	31.9	81[*d]	−
GS 0834−430	star[e]	−	12.3	105.8	$0.1 < e < 0.17$[f]
GRO J1008−57	star	−	93.5	247.5[*]	0.66[*g]
4U 1145−619	V801 Cen	B0.2III[h]	292	187[*]	large
4U 1258−61	V850 Cen	B0.7V[h]	272	132.5[*]	large
2S 1417−624	star	−	17.6	42.1	0.45
XTE J1946+274	star	−	15.8	172[i]	−
EXO 2030+375	star	−	41.7	46.03	0.41[j]
XTE J1543−568	−	−	27.1	76.6	<0.03[k]
2S 1845−024	−	−	94.3	242.2	0.88[l]
GRO J2058+42	−	−	198	110[*m]	−
A 0535−668	star	B0.5III[n]	0.07	16.7[*]	large
A 0726−26	V441 Pup	O8.5V	103.2	35[*]	−
A 1118−616	Wray 977	O9.5V	406.5	−	−
Cep X−4	star	B1V?	66.3	−	−
SAX J2239.3+6116	star	−	−	262[*o]	−

[a] Negueruela & Okazaki (2001), [b] Negueruela et al. (1999), [c] Reig et al. (2001),
[d] Corbet & Peele (2000), [e] Israel et al. (2000), [f] Wilson et al. (1997),
[g] M. Scott, priv. comm., [h] Negueruela, in prep., [i] Wilson et al., in prep.,
[j] Wilson et al. (2001), [k] in't Zand et al. (2001), [l] Finger et al. (1999),
[m] Wilson et al. (1998), [n] Negueruela & Coe (2001), [o] in't Zand et al. (2000).

where Φ is the potential defined by Eq. (10). Also shown is the distance scale corresponding to $0.1c_s P_{orb}$. This scale should be taken as an upper limit of the distance over which the disc outer radius can spread out during one orbital period, because the outflow velocity in Be discs is certainly much smaller than a few $\mathrm{km\,s^{-1}}$.

From Figs. 1 and 2a, we note that the truncation radii of the Be disc in V 0332+53 for $\alpha \lesssim 0.60$ is smaller than the critical lobe size at periastron. In other words, the Be disc never fills the critical lobe even at periastron unless the viscosity is very high ($\alpha \gtrsim 0.6$).

Table 2 gives the gap size $\Delta r = r_{crit} - r_t$, where r_{crit} is the mean radius of the critical lobe at periastron, together with stellar parameters adopted and the resulting truncation radii. Table 2 also gives lower limits for the drift timescale, $(\tau_{drift}/P_{orb})_{min}$, which is given by $(\tau_{drift}/P_{orb})_{min} \sim \Delta r/[(v_r)_{max} P_{orb}] \sim \Delta r/(0.1c_s P_{orb})$.

For a system with $(\tau_{drift}/P_{orb})_{min} > 1$, the gap is wide and the truncation is efficient, while the spread of the disc can make the truncation inefficient for a system with $(\tau_{drift}/P_{orb})_{min} < 1$.

Given that we expect the viscosity in the disc to be of the order of $\alpha \lesssim 0.1-1$, the Be disc in V 0332+53 is likely to be truncated at the 3:1 resonance radius or the 4:1 resonance radius. If the disc is truncated at the 3:1 radius, which is close to the mean critical radius at periastron [$\Delta r/a \simeq 0.03$ and $(\tau_{drift}/P_{orb})_{min} \simeq 0.56$], a small perturbation easily causes the outermost part of the disc to fall into the gravitational well of the neutron star. However, the X-ray history of this system described above suggests that the system shows no type I X-ray outburst in its normal state. Therefore, we expect that the disc in V 0332+53 has a viscosity parameter (slightly) less than 0.1 and/or a temperature (slightly) lower than $\frac{1}{2}T_{eff}$. Under those
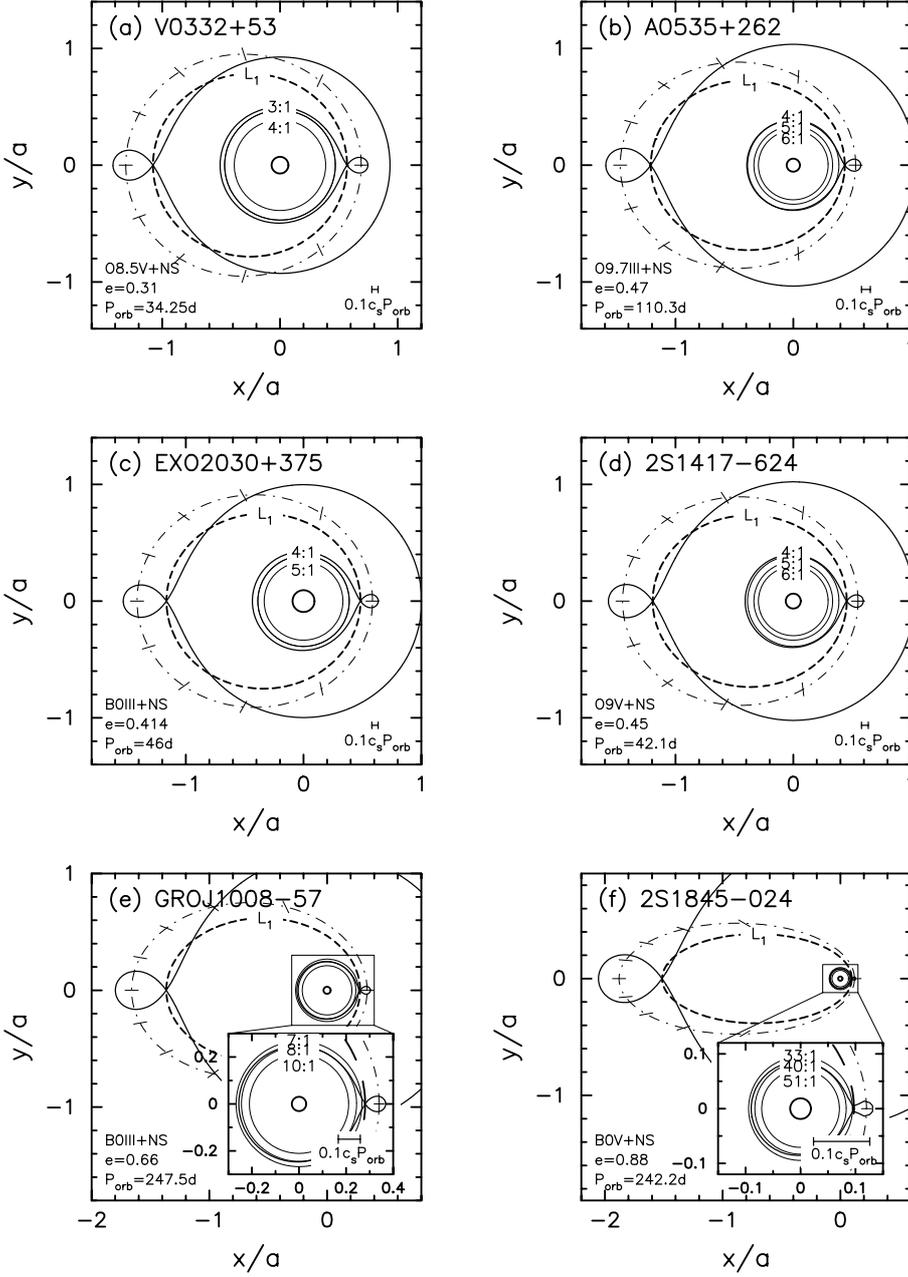
Fig. 2. Description of the orbital model adopted for each system, in the reference system centred on the Be star. The dash-dotted line represents the orbit of the neutron star. The thick dashed line represents the position of the first Lagrangian point (L_1) around the orbit. The solid thin lines are the critical lobes of the two stars at apastron and periastron (the position of the neutron star is marked with a cross). The labelled solid lines represent the locations of the n:1 commensurabilities of disc and binary orbital periods at which truncation occurs for $\alpha = 0.3$, 0.1, and 0.03 (starting from the outside). The distance scale corresponding to $0.1 c_s P_{\rm orb}$ is shown at the lower-right corner of each panel.

sconditions, it will be truncated at the 4:1 resonance radius [$\Delta r/a \simeq 0.11$ and $(\tau_{\rm drift}/P_{\rm orb})_{\rm min} \simeq 2.1$].

5.2. A 0535+262

This pulsar has an orbital period $P_{\rm orb} = 110.3$ d and an eccentricity $e = 0.47$ (Finger et al. 1996a). There is some discussion about the exact spectral type of the optical counterpart. Most authors support a giant in the range O9.5 – B0, but Wang & Gies (1998) cannot rule out a main-sequence classification based on UV spectra. Wang & Gies (1998) constrain the mass of the optical component to the range $8\,M_\odot \leq M_* \leq 22\,M_\odot$. If the object is a giant, the mass should be close to the upper limit. Taking $M_* = 20\,M_\odot$, the mass function indicates $\sin i = 0.43$, resulting in an orbital separation $a = 286\,R_\odot$.

Four type II X-ray outbursts from A 0535+262 have been observed (in 1975, 1980, 1989 and 1994). The system also displays series of type I outbursts interspersed with periods of quiescence. The length of the series or of the quiescent states is very variable. A series of three type I outbursts was observed by BATSE during 1993, followed by a type II outburst and later two weaker type I outbursts (1996a). After that, the source has been in quiescence and the optical counterpart has lost and reformed its circumstellar disc (Haigh et al. 1999).

Figure 1 shows that the Be disc in A 0535+262 is expected to be truncated at the 4:1 resonance radius ($r_t/a \simeq 0.39$) for $0.11 \lesssim \alpha \lesssim 0.40$ and at the 5:1 resonance radius ($r_t/a \simeq 0.33$) for $0.038 \lesssim \alpha \lesssim 0.11$. As shown in Fig. 2b and Table 2, the 4:1 resonance radius is slightly larger than the mean radius of the critical lobe

Table 2. Stellar parameters adopted for the systems modelled (those with known orbital parameters) and resulting critical lobe radius r_{crit}, gap size Δr, and lower limit for τ_{drift}/P_{orb} for different values of the viscosity parameter.

Name	Spectral Type	M_* ($M_\odot$)	R_* ($R_\odot$)	T_{eff} (K)	$\dfrac{r_{crit}}{a}$	$\alpha = 0.03$		$\alpha = 0.1$		$\alpha = 0.3$	
						$\dfrac{\Delta r^{\,a}}{a}$	$\left(\dfrac{\tau_{drift}}{P_{orb}}\right)^{b}_{min}$	$\dfrac{\Delta r}{a}$	$\left(\dfrac{\tau_{drift}}{P_{orb}}\right)_{min}$	$\dfrac{\Delta r}{a}$	$\left(\dfrac{\tau_{drift}}{P_{orb}}\right)_{min}$
4U 0115+634	B0V	16.0	8.0	3.0×10^4	0.46	0.13	2.8	0.08	1.7	−0.003	−
V 0332+53	O8.5V	20.0	8.8	3.4×10^4	0.50	0.11	2.1	0.03	0.56	0.03	0.56
A 0535+262	B0III	20.0	15.0	3.1×10^4	0.38	0.09	1.2	0.05	0.66	−0.004	−
EXO 2030+375	O9III	22.0	15.0	3.4×10^4	0.43	0.09	1.7	0.04	0.72	−0.04	−
	B0III	20.0	14.0	2.8×10^4	0.42	0.09	1.7	0.04	0.69	0.04	0.69
	B0V	16.0	8.0	3.0×10^4	0.41	0.12	2.0	0.08	1.4	0.03	0.45
2S 1417−624	O9V	20.0	9.0	3.4×10^4	0.40	0.10	1.8	0.06	1.1	0.01	0.18
	B1V	12.0	7.0	2.5×10^4	0.37	0.11	1.9	0.08	1.4	0.04	0.75
GRO J1008−57	B0III	20.0	14.0	2.8×10^4	0.25	0.04	0.38	0.002	0.020	−0.02	−
	B0V	16.0	8.0	3.0×10^4	0.24	0.04	0.41	−0.004	−	−0.03	−
2S 1845−024	B0V	16.0	8.0	3.0×10^4	0.084	0.014	0.13	0.001	0.013	−0.010	−
	B1V	12.0	7.0	2.5×10^4	0.081	0.015	0.15	0.004	0.038	−0.007	−

[a] $T_d = \frac{1}{2}T_{eff}$ is adopted for all models.
[b] $v_r = 0.1c_s$ is adopted as the upper limit on the radial velocity. The value is given for $\Delta r > 0$.

at periastron, while the higher resonance radii are significantly smaller than the critical lobe. The X-ray history of A 0535+262 suggests that the Be disc in this system has $\alpha \gtrsim 0.1$ and $T_d \sim \frac{1}{2}T_{eff}$ and is truncated at the 4:1 resonance radius, which will allow enough mass accretion on to the neutron star at every periastron passage to cause a type I outburst regularly (see scenario A in Fig. 3). If this is the case, a slight decrease in viscosity (and/or disc temperature) will lower the disc radius to the 5:1 resonance radius and therefore change the system to a dormant state, as has been repeatedly observed.

5.3. EXO 2030+375

This transient X-ray pulsar was discovered during a type II outburst in 1985. Since then, it has displayed tens of type I X-ray outbursts in long series separated by periods of quiescence. The orbit is characterised by $P_{orb} = 46.0\,\mathrm{d}$ and eccentricity $e = 0.41$ (Wilson et al. 2001). The optical component of this system is heavily obscured and no determination of the spectral type has been possible, but both the measured mass function and the infrared spectrum imply a spectral type earlier than about B1 (Reig et al. 1998). Coe et al. (1988) showed that the colours of the object are compatible with a B0 spectral type. The reddening $E(B-V) \simeq 3.7$ derived by Motch & Janet-Pacheco (1987) and the distance of 5.3 kpc derived by Parmar et al. (1989) from the change rates in spin period and X-ray luminosity can only be compatible if the object is a giant. However, given that the distance determination has a large uncertainty, the possibility that the object is a main-sequence star cannot be ruled out.

If the spectral type is B0III, a mass of around $M_* = 23\ M_\odot$ is expected. We have adopted a model with $M_* = 20\ M_\odot$ and $R_* = 14\ R_\odot$, in which case the mass function implies an inclination angle $i = 56°$ – this is consistent with the fact that the shape of the Hα emission lines shown by Norton et al. (1994) is typical of a Be star with moderate inclination. The orbital separation would then be $a = 150\ R_\odot$. In order to investigate the dependence of our model on the mass of the primary, we have also considered the case of $M_* = 16\ M_\odot$, $R_* = 8\ R_\odot$ (a typical B0V star).

Figure 1 shows that the difference in α_{crit} between the B0III star model and the B0V star model is small. In the B0III star model, the disc is truncated at the 4:1 resonance radius ($r_t/a \simeq 0.39$) for $0.098 \lesssim \alpha \lesssim 0.37$ and at the 5:1 resonance radius ($r_t/a \simeq 0.33$) for $0.030 \lesssim \alpha \lesssim 0.098$, while in the B0V star model the disc is truncated at the 4:1 resonance radius for $0.12 \lesssim \alpha \lesssim 0.44$ and at the 5:1 resonance radius for $0.036 \lesssim \alpha \lesssim 0.12$. We have also tried a slightly earlier spectral type, O9III, for the primary, expecting that a higher disc temperature would result in a larger truncation radius. However, the resulting radii are almost the same as those for a B0III primary, unless α is as high as 0.3 (see Table 2).

As shown in Fig. 2c, the 4:1 resonance radius is close to the mean radius of the critical lobe at periastron, while the higher resonance radii are significantly smaller than the critical lobe. Our result suggests that the observed X-ray behaviour of EXO 2030+375, which has regularly exhibited type I X-ray outbursts, favours a B0III or a O9III primary star with a disc with $\alpha \gtrsim 0.1$ and/or $T_d \gtrsim \frac{1}{2}T_{eff}$ that is truncated at the 4:1 resonance radius (see scenario A in Fig. 3).

A. Overflow Induced by Viscous Decretion
→ Regular Type I Outbursts

(a) (b) (c) (d)

B. Overflow Induced by a Density Wave
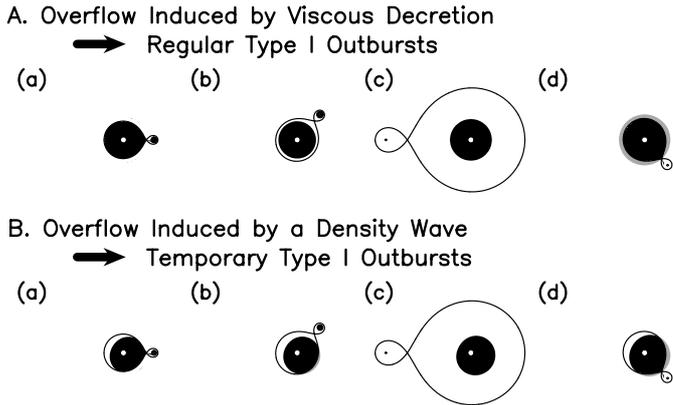→ Temporary Type I Outbursts

(a) (b) (c) (d)

Fig. 3. Scenarios for two families of type I outbursts. **A)** Regular type I outbusts induced by viscous decretion in an axisymmetric disc. **a)** Near periastron, the gas continues to overflow through the L_1 point, replenishing the accretion disc around the neutron star. (The accretion disc begins to form at panel **d)**.) **b)** At the phase at which the critical lobe radius becomes larger than the disc radius, the overflow stops and the accretion disc begins to fade. **c)** While the neutron star orbits far from periastron, the disc material drifts outward by viscous decretion. **d)** At the phase at which the critical lobe radius becomes smaller than the disc radius, the gas in the outermost part of the disc begins to overflow. The accretion disc is formed around the neutron star. **B)** Temporary type I outbursts induced by a slowly precessing density wave. **a)** If the disc is elongated roughly toward the periastron by chance, the overflow occurs around periastron. **b)** At the phase at which the critical lobe radius becomes larger than the elongated disk size, the overflow stops and the accretion disc begins to fade. **c)** Same as panel **c)** in scenario A. **d)** If the axis of elongation is still roughly toward the periastron, at the phase at which the critical lobe radius becomes smaller than the elongated disk size, the gas in the outermost part of the disc begins to overflow. The accretion disc is formed around the neutron star.

5.4. 2S 1417−624

2S 1417−624 was observed once in 1978 and was not detected again until 1994, when a type II outburst was observed by BATSE (Finger et al. 1996b). This was followed by five type I outbursts peaking near apastron.

The pulsar in this Be/X-ray transient has $P_{\rm orb} = 42.1\,{\rm d}$ and an eccentricity $e = 0.45$ (Finger et al. 1996b). The optical component of the system was studied by Grindlay et al. (1984). Owing to the low signal-to-noise of their spectra, they did not assign a spectral type, though several He II lines seem to be present in the spectrum displayed, suggesting an O-type star. Therefore we will consider again two models, one corresponding to an O9V star ($M_* = 20\,M_\odot$, $R_* = 9\,R_\odot$) and one roughly corresponding to a B1V star ($M_* = 12\,M_\odot$, $R_* = 7\,R_\odot$).

As shown in Fig. 1, the O9V star model and the B1V star model give rather different values of $\alpha_{\rm crit}$: the O9V star model results in a value for $\alpha_{\rm crit}$ similar to that in the model for A 0535+262, whereas the B1V star model gives a significantly higher value of $\alpha_{\rm crit}$. In the O9V star model, the disc is truncated at the 4:1 resonance radius

($r_{\rm t}/a \simeq 0.39$) for $0.14 \lesssim \alpha \lesssim 0.52$ and at the 5:1 resonance radius ($r_{\rm t}/a \simeq 0.33$) for $0.047 \lesssim \alpha \lesssim 0.14$, while in the B1V star model the disc is truncated at the 4:1 resonance radius for $0.34 \lesssim \alpha \lesssim 1.2$ and at the 5:1 resonance radius for $0.12 \lesssim \alpha \lesssim 0.34$. Therefore, if the optical counterpart of 2S 1417−624 is a B1V star, the disc is likely to be truncated at the 5:1 resonance radius or the 6:1 resonance radius, which is significantly smaller than the critical lobe at periastron (see also Table 2). In this case, the system is unlikely to display type I X-ray outbursts in a normal state, which is consistent with the X-ray history of this system. The O9V star model is also consistent with the X-ray history if $\alpha \lesssim 0.1$.

We interpret the sequence of type I outbursts which followed the 1994 type II outburst as a temporary phenomenon caused by a strong disturbance in the disc. After type II outbursts, the disc is expected to be strongly disturbed (see Negueruela et al. 2001, from now on Paper II, for a discussion of the possible causes of type II outbursts). Such a disc is likely to be strongly asymmetric and to be temporarily capable of fuelling the neutron star (see scenario B in Fig. 3).

5.5. GRO J1008−57

BATSE discovered GRO J1008−57 during a type II outburst in July 1993. Four weak X-ray outbursts separated by ≈248 days followed. The source is believed to emit some X-ray flux during quiescence, but no further outbursts have been observed. The optical counterpart was identified by Coe et al. (1994), but no information exists about its exact spectral type.

The orbit of this pulsar is not exactly determined. However, from the analysis of data from the BATSE experiment, a most probable orbit can be determined if the separation between type I outbursts (247.5 d) is taken to be the orbital period (M. Scott, priv. comm.). In that case, the eccentricity is $e = 0.66$ and $a_{\rm x} \sin i = 668.0 \pm 9\,{\rm lt-s}$. Again the colours of the primary are compatible with a B0 star. Therefore we use models for a B0V ($M_* = 16\,M_\odot$, $R_* = 8\,R_\odot$) and a B0III ($M_* = 20\,M_\odot$, $R_* = 14\,R_\odot$) primary.

Figure 1 shows that both stellar models give similar values of $\alpha_{\rm crit}$. In the B0III star model, the disc is truncated at the 7:1 resonance radius ($r_{\rm t}/a \simeq 0.27$) for $0.16 \lesssim \alpha \lesssim 0.36$ and at the 8:1 resonance radius ($r_{\rm t}/a \simeq 0.24$) for for $0.082 \lesssim \alpha \lesssim 0.16$. In the B0V star model, $\alpha_{\rm crit}$ becomes higher by about 20%.

From Fig. 2e, we immediately observe that the truncation radius is close to or slightly beyond the critical lobe radius at periastron unless the viscosity is very low ($\alpha \lesssim 0.03$). It should be noted that this is a typical feature in systems with high orbital eccentricity, because the resonance radii are distributed more densely for higher resonances. In other words, disc truncation is less efficient in systems with high orbital eccentricity than in systems with low or mild eccentricity. Moreover, GRO J1008−57

has a rather long orbital period. Consequently, the Be disc in this system can spread out significantly while the neutron star is far away from the Be star. These features should enable GRO J1008−57 to display type I outbursts regularly.

5.6. 2S 1845−024

2S 1845−024 has been detected by several satellites at luminosities compatible with type I outbursts (see Finger et al. 1999 for references). Between the launch of the *ComptonGRO* satellite and 1997, BATSE detected type I outbursts from 2S 1845−024 at every periastron passage. This behaviour has continued at least until 2000 (Finger, priv. comm.).

This X-ray pulsar is in a very eccentric ($e = 0.88$) and wide ($P_{\mathrm{orb}} = 242.2$) orbit (Finger et al. 1999). The optical counterpart has not been identified, but the mass function constrains it to have $M_* \gtrsim 7\ M_\odot$. Given that it has an exact orbital solution, we have modelled the system in spite of the lack of data on the optical companion. We have considered two models: one in which the primary is our typical B0V star ($M_* = 16\ M_\odot$, $R_* = 8\ R_\odot$) – adopting the parameters of a B0III star did not alter significantly the value of α_{crit} – and one with $M_* = 12.0\ M_\odot$, which is close to the value used by Finger et al. (1999) for their evolutionary model ($M_* = 11.3\ M_\odot$), and then $R_* = 7\ R_\odot$ for a typical B1V star.

As in GRO J1008−57, both stellar models make little difference in the value of α_{crit} (Fig. 1), and the truncation radius is close to or slightly beyond the critical lobe radius at periastron unless the viscosity is very low ($\alpha \lesssim 0.03$) (Fig. 2f and Table 2). With the extremely high orbital eccentricity, the disc truncation in this system is expected to be even less efficient than in GRO J1008−57. In addition, the system has a long orbital period. Therefore, it is no surprise that 2S 1845−024 has regularly shown type I outbursts without failure.

6. Discussion

In Paper I, we analysed the disc size of V635 Cas, the optical counterpart of 4U 0115+63, and found that the truncated disc size is significantly smaller than the distance to the first Lagrangian (L_1) point at periastron. Among our conclusions, we found that it is precisely this wide gap between the disc outer radius and the position of the L_1 point that prevents the system from displaying type I outbursts, by drastically reducing the accretion rate on to the neutron star. In Paper II we pointed out that, for the very same reason, the density in the disc of V635 Cas is likely to grow with time, to a point where the disc becomes optically thick and unstable to the radiation-driven warping, which seems to be at the origin of type II X-ray outbursts.

As discussed in the end of Sect. 4, we cannot expect a perfect truncation. Since the disc density is expected to decrease rapidly beyond the truncation radius, the gap size

determines how effective the disc truncation is. For systems with a wide gap, like 4U 0115+63, the truncation is so effective that the system does not show type I outbursts under normal conditions. Whatever little amount of disc material manages to overcome the tidal truncation may be easily ejected from the vicinity of the neutron star by the propeller effect (Stella et al. 1986). On the other hand, for systems with a narrow gap, the truncation will not be efficient, allowing the neutron star to capture gas from the disc at every periastron passage and display type I outbursts.

Therefore, the main purpose of this paper is to find out the general trend in the dependence of the gap size on various parameters. A second purpose is to understand why we can see two different kinds of type I outburst series: regular series of outbursts of similar intensity in some systems and short series of outbursts of varying intensity (generally associated with type II outbursts) in other systems.

6.1. Systems with large eccentricity

We have found that, in systems with high orbital eccentricity, say $e \gtrsim 0.6$, such as GRO J1008−57 ($e = 0.66$) and 2S 1417−624 ($e = 0.88$), the truncation radius of the Be disc is very close to, in the sense that $(\tau_{\mathrm{drift}})_{\mathrm{min}} \ll P_{\mathrm{orb}}$, or slightly beyond the critical lobe radius at periastron unless the viscosity is very low ($\alpha \lesssim 0.03$). Under such conditions, we expect disc truncation not to be efficient, allowing the accretion by the neutron star of enough mass at every periastron passage to cause a type I X-ray outburst regularly (see scenario A in Fig. 3). Moreover, the long orbital period of GRO J1008−57 ($P_{\mathrm{orb}} = 247.5\,\mathrm{d}$) and 2S 1845−024 ($P_{\mathrm{orb}} = 242.2\,\mathrm{d}$) will enable the Be disc in these systems to spread out significantly while the neutron star is far from periastron. This larger drift time will allow the neutron star in such systems to accrete rather more matter at periastron passage than in a system with shorter orbital period.

Among systems which we expect to be relatively well described by these conditions, we list 4U 1258−61 (GX 304−1) which displayed regular type I outbursts every 132.5 d during most of the 1970s (Corbet et al. 1986) and which has been dormant ever since because of the disappearance of the Be disc. Similarly, the behaviour of 4U 1145−619, which displays short and not very strong outbursts every ≈188 d, is indicative of a large eccentric orbit. This is supported by the lower limit $e \gtrsim 0.6$ found by Cook & Warwick (1987) from the analysis of changes in the X-ray pulse timing. Occasionally, some of the periodic outbursts from 4U 1145−619 are rather stronger (challenging the conventional definition of type II outbursts, since they cannot be described as very bright). This may be related to a large global perturbation in the Be disc, reflected in the profile of emission lines (Stevens et al. 1997). RX J0812.4−3114 has also shown a long series of type I outbursts without any intervening type II outburst

during a 4-year quasi-cycle of Be disc formation and dissipation (Reig et al. 2001). Finally SAX J2239.3+6116, which has shown several type I outbursts separated by $\approx$262 d, could also belong to this category.

One very peculiar system is A 0535−668, which has a very short orbital period ($P_{\rm orb} = 16.7$ d) and is believed to have a very high eccentricity (Charles et al. 1983). If the eccentricity is really $e \gtrsim 0.8$, the neutron star passes close to the surface of the Be star at periastron. Our model is simply not applicable to such an extreme system.

It must be noted that the observed behaviour of GRO J1008−57 is only partially compatible with the orbital model. The type I outbursts are short and not very strong, with similar luminosities, as expected for such a high-eccentricity system. However, the fact that only four have been observed and that they occurred after a type II outburst casts some doubts about the validity of the orbital solution used.

6.2. Systems with low eccentricity

It is well known that accretion discs in circular binaries with mass ratio $0.05 \lesssim q \lesssim 0.25$ are truncated at the 3:1 resonance radius by the tidally driven eccentric instability (e.g., Osaki 1996). Since the mass ratios of all Be/X-ray binaries fall within this range and the mechanism of truncation is the same for decretion discs as for accretion discs, we expect that Be discs in systems with very low orbital eccentricity, say $e \lesssim 0.2$, are also truncated at the 3:1 resonance radius. Consequently, the gap size in these systems should be much wider than in systems with larger eccentricities. Wide gaps in low-eccentricity systems will result in disc truncation being so effective that no type I outbursts should occur unless the Be disc is very strongly disturbed. Therefore it is likely that systems with very low orbital eccentricity will show only type II outbursts (and perhaps temporarily type I outbursts only when the disc is strongly disturbed).

From this point of view, GS 0834−430 is an extraordinary system, since it has rather low eccentricity (constrained to be $0.10 \lesssim e \lesssim 0.17$), but has shown a long series of type I outbursts (Wilson et al. 1997). It must be noted that the abrupt change in the separation between outbursts observed during this series is highly unusual. Moreover, the decrease in peak intensity of the outbursts along the series suggests that the Be disc was strongly perturbed at the time of the outbursts. We have to admit, however, that with the information available, our model obviously fails to explain this system.

Apart from X Per, which shows no outbursts, systems with low eccentricity are XTE J1543−568 ($e < 0.03$), which has been detected only recently and does not seem to show any clear modulation of its X-ray lightcurve at the orbital period (in't Zand et al. 2001), and 2S 1553−542 ($e < 0.09$), which was only detected once in 1975 (Kelley et al. 1983). Our interpretation of their behaviour is that these objects, because of effective disc truncation, never

display type I outbursts. Their discs therefore accumulate mass until very large perturbations develop. Only occasionally will one of these perturbations result in transfer of matter towards the neutron star, producing a bright type II outburst.

In this respect, it is necessary to insist that the physical mechanisms leading to type II outbursts are not well understood. Our model indicates that the truncated discs in Be/X-ray binaries cannot reach a steady state and will continuously grow denser. As a consequence, we expect the discs to become dynamically unstable. In the case of 4U 0115+63, which has been carefully studied (and perhaps also in A 0535+262), there is observational evidence that links big dynamical instabilities and type II outbursts (Paper II). From this observational fact, we conclude that the dynamical perturbation *somehow* results in the transfer of large amounts of material from the Be disc to the neutron star. No aspect of our model implies that such mass transfer *must* occur. It may well be that in systems with low eccentricity, dynamical instability generally leads to the collapse of the disc and its fallback on to the Be star without any substantial amount of material reaching the neutron star. If this is the case, low-eccentricity transients would be completely undetectable most of the time.

Until recently, it had been generally assumed that the orbits of Be/X-ray binaries were in general rather eccentric owing to the effect of supernova kicks. The recent determination of orbital parameters for several systems with low eccentricity (Delgado-Martí et al. 2001; in't Zand et al. 2001) may represent a challenge to this idea. If there is a selection effect against the detection of systems with low eccentricities, their population could be rather larger than previously suspected. If eccentric systems display X-ray outbursts rather more frequently than systems with low eccentricities, they would be more likely to be detected over a limited time-span (such as the time since the start of X-ray astronomy).

The orbit of the pulsar GRO J1948+32 is constrained to have an eccentricity $e \lesssim 0.25$ and an orbital period 35 d $< P_{\rm orb} < 70$ d (Chakrabarty et al. 1995). Its recent identification with the Be/X-ray transient KS 1947+300 means that it has probably displayed at least three type II outbursts between 1989 and 2001. An accurate determination of its orbit would therefore be very important in understanding whether this selection effect really exists. If its eccentricity turns out to be very low, then we will have to conclude that low-eccentricity systems may display type II outbursts as frequently as systems with moderate eccentricities. If, in contrast, its eccentricity is close to $e = 0.25$, the available data would suggest that systems with low eccentricities have less frequent type II outbursts.

A second system that could serve to understand the outbursting behaviour of systems with low eccentricities is the LMC transient EXO 0531.1−6609. Based on a statistical parameter study, Dennerl et al. (1996) conclude that this pulsar ($P_{\rm s} = 13.7$ s) is likely to have an orbital period $P_{\rm orb} = 25.4$ d and low eccentricity $e \approx 0.1$. EXO 0531.1−6609 has displayed at least three relatively

bright outbursts extending over more than one orbital period (i.e., type II) in 1983, 1985 and 1993, but also seems to have been occasionally detected by *ROSAT* at a low X-ray luminosity close to periastron. Obviously confirmation of the orbital parameters is necessary before any conclusions can be drawn.

6.3. Systems with moderate eccentricity

Four systems (V 0332+53, A 0535+262, EXO 2030+375, 2S 1417−624) among the six Be/X-ray binaries discussed in the previous section have moderate eccentricity. As we have already seen, the model results for these systems turn out to depend on rather subtle details, in contrast to the much more robust predictions for systems with high eccentricity and very low eccentricity. Since the spacing of the candidate truncation radii is rather larger in these systems than in high-eccentricity Be/X-ray binaries, a small difference in the system parameters (orbital, stellar or disc parameters) can produce significantly different X-ray behaviour, depending on whether the resulting truncation radius happens to be close to the critical lobe radius or not. Therefore we have not tried to present general predictions for these systems. Instead, in the previous section we have estimated the truncation radii and system parameters by comparing the model results with the observed X-ray behaviour. In this sense, the current status of our model can still be considered to be "at the qualitative level" for mildly eccentric systems. We estimated that Be discs in V 0332+53, A 0535+262 in the X-ray active state, and EXO 2030+375 are truncated at the 4:1 resonance radius, while those in A 0535+262 in quiescence and 2S 1417−624 are truncated at the 5:1 resonance radius. The smaller truncation radius for A 0535+262 in quiescence might suggest a slightly lower disc temperature in this state than in the X-ray active state. To understand more about these systems, we need a more sophisticated model for the interaction between the Be disc and the neutron star.

It is worth noting, however, that all the intermediate eccentricity Be/X-ray binaries known have displayed type II outbursts. This is the case for the four systems listed here and also for GRO J1750−27, a presumably very distant source (with $P_{\rm orb} = 29.8\,{\rm d}$ and $e = 0.36$) which has only been detected once during a type II outburst (Scott et al. 1997). This fact suggests that disc truncation is generally rather effective for orbital eccentricities $e \lesssim 0.5$, even though the discs in A 0535+262 and EXO 2030+375, under certain conditions, seem to reach a state in which truncation is not so effective and long series of type I outbursts occur (not surprisingly, these two systems have the largest eccentricities and longest periods among the five known).

Though systems with moderate eccentricities dominate the sample of Be/X-ray binaries for which orbital solutions exist at present, it is important to note that at least two important selection effects may have been at work in the definition of this sample:

1. Be/X-ray binaries with moderate eccentricities are much more likely to display type II outbursts than systems with high eccentricities and perhaps also more likely than systems with low eccentricities (see above). Their detection as new X-ray sources is therefore easier.

   Among Be/X-ray binaries, 4U 0115+63 has been by far the most active during the era of X-ray astronomy, displaying no fewer than 13 type II outbursts between 1969 and 2000. From the estimates in Paper II, we come to the conclusion that the enhanced activity of 4U 0115+63 is not owing to causes intrinsic to the Be star (such as a particularly high mass-loss rate) and therefore it is likely to be connected to its orbital parameters.

   The combination of moderate eccentricities and relatively close orbits results in rather efficient disc truncation, which allows the storage of matter in the Be disc, but at the same time results in gaps which are not too wide and may permit the transfer of part of this material to the neutron star under certain conditions;

2. Once the system has been detected, a relatively close orbit may be solved from X-ray observations spanning one single type II outbursts, while in a larger orbit a much longer baseline will be needed (only recently, permanent coverage over long time-spans with BATSE and *RXTE* has allowed the solution of some orbits with $P_{\rm orb} \gtrsim 50\,{\rm d}$).

Because of these two factors, judgement on what sort of X-ray behaviour is more "typical" among Be/X-ray binaries has to be suspended until a rather larger sample is known.

6.4. Predictions

Many of the systems under discussion have shown short series of irregular type I outbursts. This category includes 4U 0115+63, V 0332+53, 2S 1417−624 and presumably GRO J2058+42, for which no orbital solution exists. These series have generally occurred in connection with type II outbursts (in most cases, after a type II outburst). The behaviour of A 0535+262 has also been similar on some occasions. As mentioned in Sect. 5.4, we expect that the temporary type I outbursts occur in systems in which the Be disc has a radius significantly smaller than (but not too small compared to) the critical lobe radius under normal conditions. These systems will be mostly mildly eccentric systems. If the Be disc in such a system is strongly disturbed by, e.g., radiation-driven warping and/or a global density wave, it will become strongly asymmetric. Then, if the disc elongation is roughly in the periastron direction, the disc will be capable of fuelling the neutron star and cause a sequence of type I X-ray outbursts (see scenario B in Fig. 3). Since GRO J2058+42 has apparently displayed

similar behaviour, we expect it to have moderate eccentricity.

A relationship between the X-ray behaviour and the orbital eccentricity stems naturally from the truncated disc model. In general, the presence of regular type I X-ray outbursts indicates that the truncated disc around the Be star has a radius close to or slightly larger than the critical lobe radius at periastron. Such a situation occurs in all systems with highly eccentric orbit, some mildly eccentric systems, and no systems with low eccentricity (GS 0834−430 may be an exception). In a rough sense, the lower the orbital eccentricity, the wider the gap between the truncation radius and the critical lobe radius and therefore the more difficult the accretion on to the neutron star.

A source which hardly fits in this picture is A 0726−26. This persistent low-luminosity X-ray source which has never shown outbursts displays, however, a 35-d modulation in its X-ray emission (Corbet & Peele 1997). If this is the orbital period, A 0726−26 presents very few similarities to other Be/X-ray binaries, falls very far away from the $P_{\rm s}/P_{\rm orb}$ correlation and cannot be explained by our model.

Among the Be/X-ray binaries with known counterparts, two systems have been very rarely detected as X-ray sources, and even then they have been relatively weak. A 1118−616 has only been detected twice, in 1975 and in 1991. In both cases, the outburst was long, but not very bright. Cep X-4 has been detected four times between 1972 and 1997. All the outbursts have also been relatively weak. The two sources display permanent low-luminosity X-ray emission when not in outburst. In both cases, the counterparts present evidence for very large circumstellar discs. Therefore we expect these two systems to have wide and not very eccentric orbits.

We note that even at the qualitative level our model has some predictive power and can thus be tested. Finding out, for example, that RX J0812.4−3114 has an orbital eccentricity close to zero or that A 1118−616 has a very eccentric orbit would certainly force us to reconsider most of the conclusions reached here.

7. Concluding remarks

We have applied the resonantly truncated disc model developed by Negueruela & Okazaki (2001) to six systems which have displayed type I X-ray outbursts. We have found that the model naturally explains the X-ray behaviour of these systems and that all systems are consistent with having a similar viscosity parameter $\alpha \sim 0.1$.

In our model, regular type I X-ray outbursts occur in all systems with high orbital eccentricity, some mildly eccentric systems, and no systems with low eccentricity. On the other hand, systems which have temporarily shown type I outbursts will be mostly mildly eccentric systems, in which the Be disc temporarily becomes strongly asymmetric.

In the scenario presented here for Be/X-ray transients, disc material can reach the neutron star only via the L_1 point, and will therefore have a not very high velocity relative to the neutron star. Since such a flow (which may be said to represent the disc version of Roche-lobe overflow) carries high angular momentum, an accretion disc may temporarily be formed around the neutron star during each X-ray outburst. Such temporary accretion discs have probably been observed in 2S 1845−024 (Finger et al. 1999), but extensive searches have failed to detect them around A 0535+262 (Motch et al. 1991). The accretion flow in the vicinity of the neutron star (i.e., once it has left the effective Roche lobe of the Be star) should be modelled in detail in order to understand if such discs should always form and to predict the X-ray lightcurves derived from our scenario.

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