

The black hole at the galactic center as a possible retro-lens for the S2 orbiting star

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Abstract. Holz & Wheeler (2002) have recently proposed that a Schwarzschild black hole may act as a retro-lens which, if illuminated by a powerful light source, deflects light ray paths to large bending angles and a series of luminous arcs (or rings in the case of aligned objects) centered on the black hole may form. Obviously, the most convenient geometry to get retro-lensing images would be that of a very bright star close to a massive black hole, say the putative $\sim 4 \times 10^6 M_\odot$ black hole at the galactic center. Recent observations of the galactic center region in the *K*-band have revealed the presence of a very bright main sequence star (labelled S2) with mass $\sim 15 M_\odot$ orbiting at close distance (130–1900 AU) from Sgr A*. The relatively vicinity of S2 to the central massive black hole may offer a unique laboratory to test the formation of retro-lensing images. The next generation of space-based telescopes in the *K*-band (like NGST) may have high enough limiting magnitude necessary to observe such retro-lensing images.

Key words. gravitation – gravitational lensing

1. Introduction

It is well known that a dark object moving close to the source-observer line of sight acts as a gravitational lens, causing the formation of two or more images of the source. If the images can not be resolved, the lensing phenomenon results in an overall source image magnification, to which one refers as gravitational microlensing event. However, the geometry of the lensing phenomena may also be different with the observer (source) in between the source (observer) and lens line of sight.

Recently, Holz & Wheeler (2002) have proposed that a Schwarzschild black hole may act as a retro-lens (or retro-MACHO) that deflects the source light ray paths to so large bending angles that emerging photons may reach the observer (in the middle between the source and the lens). The shape and the magnitude of the formed images (arcs, becoming rings in the perfect alignment case) strongly depends on the mass of the black hole and on the relative distances between the observer, the lens and the source. In particular, in the perfect alignment case, after π rotations around the black hole, photons if collected form a series of circular rings. More recently, the generalization of the retro-lensing effect due to Kerr black holes has been analyzed by De Paolis et al. (2003).

Holz & Wheeler (2002) considered the Sun as the light source which emits photons towards a stellar black hole at distance D_L and then evaluated the magnitude of the formed

retro-lensing images. In the most suitable case (i.e. perfect alignment between the source, the observer and the lens) the maximum distance at which the retro images can be seen by an instrument with limiting magnitude \bar{m} is

$$D_L = 0.02\text{pc} \times \left[10^{(\bar{m}-30)/2.5} (M_{\text{BH}}/10 M_\odot)^2 \right]^{1/3}, \quad (1)$$

for an assumed image baseline of magnitude $m = 30$.

However, since we do not expect massive black holes lying so close to the Earth, we can say that making a survey for retro-MACHOs illuminated by the Sun might not be a successful strategy.

A better idea is to look towards well known black holes, for example at the galactic center which is thought to host a super massive black hole (Sgr A*) with mass $M_{\text{BH}} \simeq 4.07 \times 10^6 M_\odot$ (Ghez et al. 2003). Moreover, to have a chance to really detect retro-lensing images, one should also consider a very bright star close to Sgr A* as the light-source.

Indeed, if the Sun is the light source and Sgr A* is the considered black hole, the magnitude of the formed absorbed retro-images turns out to be $m_V \simeq 100$ ($\simeq 47$ if absorption in the *V* band is not taken into account).

Nowadays, large ground and space-based telescopes allow to obtain an unprecedented view of the galactic center, in terms of both sensitivity and angular resolution, so that individual stars can be monitored within $1''$ from the galactic center. In this respect, Ghez et al. (2003) have announced the observation of a bright star, labelled S2, orbiting in close proximity to the massive black hole at the center of the Galaxy.

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The aim of this letter is to show that the binary system S2-Sgr A* may offer a unique opportunity to study the retro-lensing phenomenon proposed by Holz & Wheeler (2002), taking advantage of both the large black hole mass and the small separation of the binary components.

2. S2-Sgr A* binary system

Very recently, Ghez et al. (2003) have observed a star orbiting in close proximity to the galactic center massive black hole. The orbital parameters of the binary system are given in Table 1. The star, which has been labelled as S2, with mass $M_{S2} \sim 15 M_{\odot}$, appears to be a main sequence star, probably an O8-B0 dwarf with surface temperature $\sim 3 \times 10^4$ K and bolometric luminosity $\sim 10^3 L_{\odot}$. In order to evaluate the magnitude of the retro-image we need to calculate the magnification factor μ by considering the geometry of the S2-Sgr A* binary system. Referring to Fig. 1, we consider the S2 source which is moving around the black hole (at the center of the reference frame) on an elliptic orbit with semi-major axis a and eccentricity e . Let be i the inclination angle of the orbital plane with respect to the observer (lying the $X - Z$ plane), and define the misalignment angle β as the angle between the source-observer line \vec{SO} and the observer-lens line \vec{LO} , as measured at the observer. Hence, the misalignment angle β is given by

$$\beta = \cos^{-1} \left[\frac{\vec{SO} \cdot \vec{LO}}{D_{OS} D_{OL}} \right], \quad (2)$$

where D_{OL} and D_{OS} are the observer-lens and the observer-source distances, with

$$D_{OS}(t) = \left(D_{LS}^2 + D_{OL}^2 + 2D_{LS}D_{OL} \cos \delta \right)^{1/2}. \quad (3)$$

Here, $D_{OL} \simeq 8$ kpc and D_{LS} is the distance between the lens and the source, i.e. for the considered elliptic orbit

$$D_{LS}(t) = \frac{a(1 - e^2)}{1 + e \cos \phi(t)}, \quad (4)$$

Table 1. The masses M_{BH} and M_{S2} of the galactic center black hole and S2 orbiting star are given. The remaining orbital parameters are the S2 star radius R_{S2} , the semi-major axis a , the eccentricity e , the orbital period P and the inclination angle i . Data are taken from Ghez et al. (2003).

S2-Sgr A* orbital parameters	
M_{BH}	$4.07 \times 10^6 M_{\odot}$
M_{S2}	$15 M_{\odot}$
R_{S2}	$5.8 R_{\odot}$
a	4.87×10^{-3} pc
e	0.87
P	15.78 yr
i	47.3 deg

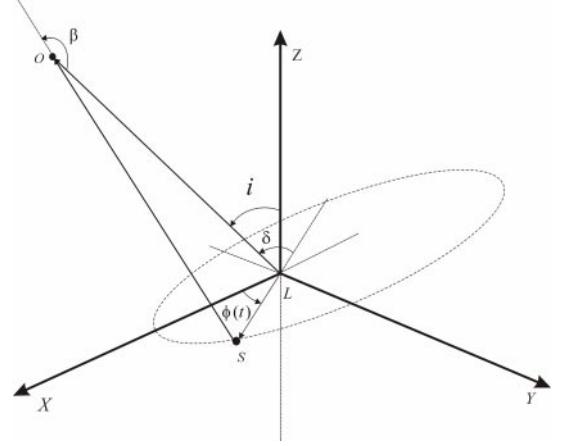


Fig. 1. The geometry of the S2-Sgr A* binary system retro-lensing phenomenon: photons are emitted by the source S (the star S2), moving on an elliptic orbit (on the $X - Y$ plane) around the central galactic black hole in L . The emitted photons go towards the lens, which lies in one of two foci of the ellipse, and after deflection by the angle δ reach the observer in O .

being $\phi = \omega t + \phi_0$ and $\omega = [G(M_{BH} + M_{S2})/a^3]^{1/2}$. The angle δ appearing in Eq. (3) is the deflection angle, as measured at the lens, needed for a source photon to reach the observer, i.e.

$$\delta(t) = \cos^{-1} \left[\frac{\vec{SL} \cdot \vec{LO}}{D_{LS} D_{OL}} \right] = \pi - \cos^{-1} (\cos \phi \sin i). \quad (5)$$

Therefore, the angle β turns out to be

$$\beta(t) = \pi - \cos^{-1} \left[\frac{D_{OL} + D_{LS} \cos \delta}{D_{OS}} \right]. \quad (6)$$

In the case of the S2 star, photons retro-lensed by Sgr A* would form two images, of angular extent $\Delta\Theta \simeq 2 \tan^{-1} (R_{S2}/D_{OS} \sin \beta)$, being R_{S2} the S2 radius. Since lensing conserves the surface brightness, the total image amplification μ is given by the ratio between the fraction of sky covered by each image to that covered by the source disk as seen by the observer.

Thus, following Holz & Wheeler (2002), one gets for the brighter (B) and fainter (F) image the amplification factors

$$\mu_{B,F} \simeq \left[b_o^2(\Theta) - b_i^2(\Theta) \right] \tan^{-1} \left(\frac{R_{S2}}{D_{OS} \sin \beta} \right) \frac{D_{OS}^2}{\pi D_{OL}^2 R_{S2}^2}, \quad (7)$$

where, for a given deflection angle Θ , $b_o(\Theta)$ and $b_i(\Theta)$ are the photon impact parameters corresponding to the inner and outer radii of the images, respectively. Finally, the total amplification results to be

$$\mu = \mu_B + \mu_F. \quad (8)$$

It has been shown by Chandrasekhar (1983) that, for a Schwarzschild black hole and in the case of large values of Θ , the photon impact parameter is

$$b(\Theta) = b_c + b_d e^{-\Theta}, \quad (9)$$

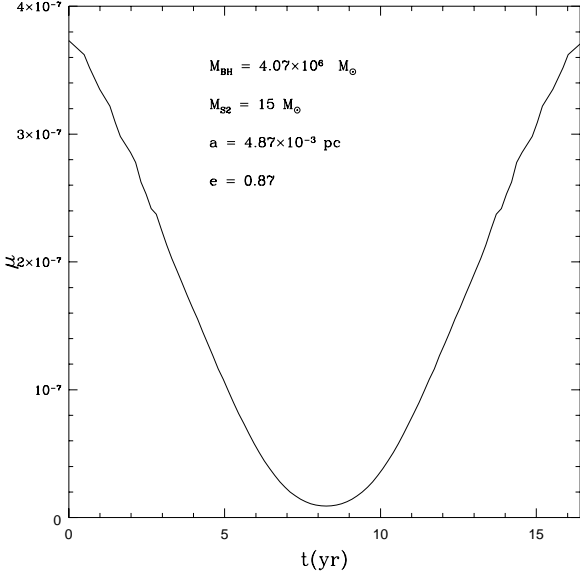


Fig. 2. The amplification factor μ as a function of time t is shown. The calculation has been made by considering the S2-Sgr A* orbital parameters given in Table 1. Note that the periodic behavior of the curve depends on the S2 orbital motion.

where

$$b_c = 3\sqrt{3}\frac{GM_{\text{BH}}}{c^2},$$

$$b_d = 648e^{-\pi}\sqrt{3}\frac{GM_{\text{BH}}}{c^2}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2. \quad (10)$$

Since the angular extent of the S2 disk with respect to Sgr A* is $\alpha \approx R_{\text{S2}}/D_{\text{LS}}(t)$, the bending angles for the brighter and fainter images are $\Theta_{\text{B}} = \delta \mp \alpha$ and $\Theta_{\text{F}} = 2\pi - \delta \mp \alpha$, respectively.

Consequently, by using Eqs. (7)–(10), it is possible to evaluate, for each position of the star S2 along its orbit around Sgr A*, the total amplification factor μ of the retro-lensing images. The obtained result is shown in Fig. 2. It is evident the periodic behavior (with period P) of the amplification factor μ , the upper value of which ($\sim 3.75 \times 10^{-7}$) corresponds to the periastron distance of S2.

The luminosity of the formed arcs is $L_\lambda = \mu L_\lambda^{\text{S2}}$, being L_λ^{S2} the S2 luminosity in the electromagnetic band centered at the wavelength λ . It follows that the image magnitude is

$$m_\lambda = m_\lambda^{\text{S2}} - 2.5 \log \mu, \quad (11)$$

being μ independent on the wavelength. For S2 in the periastron position ($\phi = 0$ and $D_{\text{LS}} \approx 130$ AU) and for the inclination angle $i = 47.3^\circ$, from Fig. 2 the maximum allowed amplification factor is $\mu_{\text{max}} \approx 3.75 \times 10^{-7}$ so that the expected minimum image magnitude results to be $m_{\lambda \text{ min}} \approx m_\lambda^{\text{S2}} + 16.1$.

Then, as a consequence of the S2 orbital motion the amplification factor μ decreases until the minimum value $\mu_{\text{min}} \approx 1.5 \times 10^{-8}$ is reached, at the furthest distance from Sgr A*, corresponding to the maximum image magnitude $m_{\lambda \text{ max}} \approx m_\lambda^{\text{S2}} + 19.6$.

Clearly, due to the absorption processes along the line of sight, the previous values have to be increased by the λ -band extinction factor A_λ .

The S2 star has been observed in the K -band (centered at $2.2 \mu\text{m}$) with an unabsorbed magnitude of $m_K^{\text{S2}} = 13.9$ (Ghez et al. 2003). Taking into account the infrared extinction factor $A_K \approx 3.3$ towards the galactic center (Rieke et al. 1989), from Eq. (11) the expected absorption corrected K -magnitude of the retro-lensing images¹ is

$$m_K = 17.2 - 2.5 \log \mu, \quad (12)$$

which for the minimum and maximum distances of S2 from Sgr A* (see Fig. 2), gives the values $m_{K \text{ min}} \approx 33.3$ and $m_{K \text{ max}} \approx 36.8$, respectively.

Obviously, in order to detect such retro-images, we need instrument with limiting magnitude $\bar{m} \geq m_\lambda^{\text{S2}} + A_\lambda - 2.5 \log \mu$ in which A_λ is the λ -band extinction factor.

For the sake of completeness, we mention that assuming the existence of a star (like S2 and with the same orbital parameters) perfectly aligned with the Earth-Sgr A* line of sight ($\phi = 0$, $i = \pi/2$ and $\beta = \pi$), the amplification factor and the corresponding ring magnitude result to be $\mu \approx 8 \times 10^{-4}$ and $m_\lambda \approx m_\lambda^{\text{S2}} + 7.74$, respectively. In particular, in the K -band, after correction for the absorption, we get a ring magnitude of $m_K \approx 21.6$, close to the present day instrument capabilities.

In this case the required instrument limiting magnitude to observe the retro-lensing images (rings in this case) has to satisfy the relation

$$\bar{m} \geq m_\lambda^* + A_\lambda - 2.5 \log \left[\left(\frac{M}{M_\odot} \right)^2 \left(\frac{R_\odot}{R_S} \right) \frac{D_{\text{OS}}^2}{D_{\text{LS}} D_{\text{OL}}^2} \right]. \quad (13)$$

3. Discussion

Retro-lensing phenomena, i.e. deflections by black holes of the light ray paths to large bending angles, have been recently proposed (Holz & Wheeler 2002) as a new way to search for black holes². However, if one considers the Sun as the light source, the limiting distance D_{L} within which a Schwarzschild black hole may show observable effects is given by Eq. (1), which assuming $M = 10 M_\odot$ and $\bar{m} = 30$ gives $D_{\text{L}} \approx 0.02$ pc. Consequently, due to the lack of any massive black holes within this distance, making a survey for retro-MACHOs illuminated by the Sun might not be a successful strategy.

A better chance to detect retro-lensing images should be looking towards the direction of well known massive black holes such as the $\approx 4.07 \times 10^6 M_\odot$ one (Sgr A*) at the galactic center.

¹ Since S2 has been classified as a O8-B0 main sequence star (Ghez et al. 2003), one expects that the optical band absolute magnitude is $M_V = -4.0$ (Allen 2000) implying, for an assumed observer-S2 distance of 8 kpc, the apparent V -magnitude $m_V^{\text{S2}} \approx 10.6$. Hence, the expected unabsorbed ring magnitude is in the range 26.7–30.2, depending on the S2 orbital position. Unfortunately, the ratio of the K to V extinction factor is $A_K/A_V \approx 0.112$ (Rieke & Lebofsky 1985) so that the V -band ring magnitude has to be correct for absorption by the factor $A_V \approx 29.5$. This implies that the S2 retro-image has magnitude in the range $56.2 \leq m_V \leq 59.7$, i.e. is much fainter than in the K -band.

² A somehow related idea has been proposed by Capozziello et al. (1997) who considered the effect of defocusing gravitational lensing.

We have shown that the recently observed bright star S2, shining the galactic center black hole, may produce retro-lensing images with apparent magnitude given by Eq. (11) in which m_{λ}^{S2} is the S2 star magnitude in the λ -band and μ the amplification factor given by Eq. (8). By taking into account the λ -band extinction A_{λ} , these images may be observed by an instrument with limiting magnitude $\bar{m} \geq m_{\lambda}^{S2} + A_{\lambda} - 2.5 \log \mu$ which gives, in the K -band, $\bar{m} \approx 33.3$ at the periastron distance of S2. Since the photon impact parameter values b_o and b_i are very close to the Schwarzschild black hole radius value $R_{Sch} \approx 1.2 \times 10^{12}$ cm, the retro-lensing images form at $\approx 20 \mu\text{arcsecs}$ from Sgr A* implying that extremely high angular resolution are required to see the details of the retro-images.

As we have seen, the expected retro-lensing image magnitude turns out to be $\bar{m} \approx 33$ in the K -band for S2 in the pericenter position. Of course, present instrumentations have not the right capability to detect such faint images. SIRTF (Space Infrared Telescope Facility), a space based telescope operating in the electromagnetic band 3–180 μm (National Research Council 2001), will allow to detect, with an integration time of 3 hours, a flux $\Phi \approx 10^{-15}$ erg $\text{cm}^2 \text{s}^{-1}$ corresponding to a limiting magnitude of $\bar{m} \approx 22$. Hence, SIRTF has not the required sensitivity to detect the S2 retro-images in the K -band. The attainable limiting magnitude will increase with the Next Generation Space Based Telescope (NGST) which, planned to work in the wavelength range 0.6–27 μm (National Research Council 2001), is expected to have a sensitivity at least 3 order of magnitude (at 2.2 μm) better than SIRTF. In fact, the energy flux (in a band centered at 2.2 μm) that NGST is able to detect, within an integration time of 3 hours, is $\Phi \approx 2 \times 10^{-19}$ erg $\text{cm}^2 \text{s}^{-1}$ which corresponds to a limiting magnitude $\bar{m} \sim 32$. This limiting magnitude is very close to that necessary to observe S2 retro-lensing images that could be effectively observed by NGST increasing the integration time to about 27 hours.

We have also to mention that, in addition to look towards Sgr A* in the K band, another possibility to observe retro-lensing images in the future may be offered by X-ray observations. Indeed, due to X-ray interferometry from space, the next generation of X-ray telescope facilities may reach an angular resolution down to 0.1 μarcsec (see e.g. MAXIM 2003 and Constellation X 2003). Clearly, the problem is the existence

of a bright enough X-ray source closely shining the Sgr A* black hole (S2, being a main sequence star, does not emit substantially in the X-ray band). Indeed, the galactic center region hosts a large number of X-ray emitting neutron stars or accreting black holes most of which are detected in the energy range 2–10 keV with a luminosity 10^{32} – 10^{35} erg s^{-1} (Wang et al. 2002; Pessah & Melia 2003). Therefore, in the future the detection of retro-lensing images in the X-ray band may be a challenging possibility.

A natural question that arises is what changes if the black hole at the galactic center is a Kerr black hole. Recently, the generalization of the retro-lensing effect due to rotating Kerr black holes has been analyzed by De Paolis et al. (2003). Apart a minor effect in changing the value of the magnitude of the retro-lensing images, the black hole spin produces a deformation of the image shape since co-rotating and counter-rotating photons have closer and further impact parameters, respectively (Peter 1976; Bray 1986). The analysis of the retro-image shape may therefore allow to specify the geometry of the system and measure the spin parameter of the black hole.

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