A&A 416, 437–440 (2004)
DOI: 10.1051/0004-6361:20034455

Astronomy & Astrophysics

Research Note

Pair production by photons in a hot Maxwellian plasma

E. Haug⋆

Institut für Astronomie und Astrophysik, Universität Tübingen, 72076 Tübingen, Auf der Morgenstelle 10, Germany

Received 6 October 2003 / Accepted 22 December 2003

Abstract. The production of electron-positron pairs by photons in the Coulomb field of electrons and positrons (triplet production) in hot thermal plasmas is investigated. The pair production rate for this process is calculated as a function of the photon energy and compared with the rate of photon-nucleus pair production for semirelativistic and relativistic plasma temperatures.

Key words. gamma rays: theory – plasmas

1. Introduction

In many astrophysical objects such as active galactic nuclei, γ-ray burst sources, or stellar black-hole binaries, a population of relativistic electrons is required to explain observations of radiation emission. Generally the energy distribution of these electrons is assumed to have a power-law form. Sometimes, however, the observational data suggest that the electron spectrum be a Maxwellian rather than a power law. Hard X-ray spectra of black-hole binaries in their hard states and of Seyfert galaxies have been modeled by thermal Comptonization, giving strong evidence that these objects have Maxwellian electron distributions (Johnson et al. 1997; Zdziarski et al. 2000; McConnell et al. 2002). Such hot plasmas may originate from collisions of large gaseous masses with high bulk velocities of the order of $0.1\,c$ which lead to the formation of shocks and approximate energy equipartition (Blandford & McKee 1976).

In a relativistic plasma strong X- and γ-radiation is generated by various mechanisms: synchrotron emission, inverse Compton scattering, and bremsstrahlung. The energetic photons, in turn, produce electron-positron pairs in scatterings off nuclei, electrons, and other photons. The production mechanisms of electron-positron pairs have been investigated by many authors in connection with astrophysical problems (e.g., Bisnovatyi-Kogan et al. 1971; Lightman 1982; Svensson 1982, 1984). In particular, the pair production arising from the interaction of photons with the Coulomb field of electrons or positrons ($\gamma-$e pair production, triplet production) has been studied by Zdziarski (1985), Mastichiadis et al. (1986, 1994), Mastichiadis (1991), and Dermer & Schlickeiser (1991), mostly in connection with electron energy losses by triplet pair production in photon fields. The threshold for $\gamma-$e pair production in the rest system of the electrons is $(h\nu)_{\mathrm{t}} = 4mc^2 \approx 2$ MeV. If, however, the electron energy is sufficiently large, the high velocity of the electrons results in pair production even by photons with energies far less than $(h\nu)_{\mathrm{t}}$. On the other hand, pair production in the Coulomb field of protons and other nuclei begins only at the threshold $2mc^2 \approx 1$ MeV as long as the target particles can be considered stationary. Therefore, under certain conditions $\gamma-$e pair production dominates relative to $\gamma-$p pair production. Actually, triplet production is a limiting factor for the transparency of the metagalaxy to very energetic photons (Bonometto et al. 1974). In the present paper the electrons are assumed to have a relativistic Maxwell-Boltzmann energy distribution with a characteristic dimensionless temperature $\Theta = k_{\mathrm{B}}T/mc^2$, where k_{B} is the Boltzmann constant and mc^2 the electron rest energy. The production rates for these processes are calculated as a function of the photon energy for a range of semirelativistic and relativistic plasma temperatures.

In the following, relativistic units will be used, i.e., the energies of electrons, positrons, and photons are expressed in units of the electron rest energy mc^2, and the momenta in units of mc, unless specified otherwise. Then the energy-momentum relation of electrons and positrons reads $\epsilon^2 = p^2 + 1$ where ϵ is the total energy including the rest energy.

⋆ e-mail: haug@tat.physik.uni-tuebingen.de

2. Pair production rates

2.1. γ–e pair production

The rate of triplet production by photons of the dimensionless energy $k = h\nu/mc^2$ in a plasma with electron plus positron number density n_e is given by

$$\frac{dN_e(k)}{dt} = n_e c \int (1 - \beta\cos\theta)\, \sigma_t(\kappa)\, f(p)\, d^3p. \quad (1)$$

Here $\beta = p/\epsilon$ denotes the electron or positron velocity in units of the speed of light, θ is the angle between the incident photon and target electron or positron, $\sigma_t(\kappa)$ is the total cross section for pair production in the field of an electron (Haug 1975, 1981),

$$\kappa = k(\epsilon - p\cos\theta) \quad (2)$$

is the invariant product of the energy-momentum four-vectors of photon and electron, ϵ is the total energy of the electron in units of mc^2, $p = \sqrt{\epsilon^2 - 1}$ is the electron momentum in units of mc, and

$$f(p) = \frac{e^{-\epsilon/\Theta}}{4\pi\Theta K_2(1/\Theta)} \quad (3)$$

is the relativistic Maxwell-Boltzmann distribution normalized to unity, $\int f(p)\, d^3p = 1$; $K_2(x)$ denotes the modified Bessel function.

There are two methods of evaluating the triplet production rate (1). Both of them require only a single numerical integration. The first one utilizes Weaver's (1976) general theory of two-body reaction rates in a relativistic plasma. For monoenergetic photons and electrons with a Maxwell-Boltzmann distribution the result is

$$\frac{dN_e(k)}{dt} = \frac{n_e c}{2k^2 K_2(1/\Theta)} \int_4^\infty \kappa\sigma_t(\kappa) \exp\left\{-\frac{1}{2\Theta}\left(\frac{\kappa}{k} + \frac{k}{\kappa}\right)\right\} d\kappa. \quad (4)$$

The total cross section for triplet production, $\sigma_t(\kappa)$, is a relativistic invariant. It can be obtained from the doubly differential cross section (Haug 1975) by two numerical integrations. The result was fitted to simple analytical functions by Haug (1981). If the photons have an energy distribution $n_\gamma(k)$, the triplet production rate is calculated by a second integration over k. The rate of this type has been given by Svensson (1984) for the case of a Wien photon distribution and by Zdziarski (1985) for the more general case of a power-law energy distribution with exponential cutoff. Both times the integration over k can be performed analytically using the formalism of Weaver (1976).

The second method to calculate $dN_e(k)/dt$ utilizes the fact that the analytical approximations of $\sigma_t(\kappa)$ (Haug 1981) can be integrated in closed form. Substituting Eq. (3) into Eq. (1) and using the relations

$$\begin{aligned} 1 - \beta\cos\theta &= \kappa/(\epsilon k), && (5)\\ d^3p &= 2\pi p^2\, dp\, d(\cos\theta) = 2\pi\epsilon p\, d\epsilon\, d(\cos\theta) &&\\ &= -2\pi(\epsilon/k)\, d\epsilon\, d\kappa, && (6) \end{aligned}$$

one gets

$$\frac{dN_e(k,\Theta)}{dt} = \frac{\alpha r_0^2 c n_e}{2k^2\Theta K_2(1/\Theta)} \int_{\epsilon_0}^\infty d\epsilon\, e^{-\epsilon/\Theta} \int_{\kappa_0}^{k(\epsilon+p)} d\kappa\, \kappa\, \frac{\sigma_t(\kappa)}{\alpha r_0^2}, \quad (7)$$

where $\alpha \approx 1/137$ is the fine-structure constant and r_0 is the classical electron radius. The minimum value of κ is given by

$$\kappa_0 = \mathrm{Max}\{4, k(\epsilon - p)\}, \quad (8)$$

and the minimum electron energy required to generate a pair by a photon of energy k is

$$\epsilon_0 = \begin{cases} 2/k + k/8 & \text{if } k \le 4; \\ 1 & \text{if } k \ge 4. \end{cases} \quad (9)$$

The latter two relations result from the kinematic condition $\kappa = k(\epsilon - p\cos\theta) \ge 4$. Since $\epsilon - p < 1$, $\kappa_0 = 4$ for $k \le 4$. Defining the function

$$S(\epsilon, k) = \int_{\kappa_0}^{k(\epsilon+p)} \kappa\, \frac{\sigma_t(\kappa)}{\alpha r_0^2}\, d\kappa, \quad (10)$$

the pair production rate gets the form

$$\frac{dN_e(k,\Theta)}{dt} = \frac{\alpha r_0^2 c n_e}{2k^2\Theta K_2(1/\Theta)} \int_{\epsilon_0}^\infty S(\epsilon, k)\, e^{-\epsilon/\Theta}\, d\epsilon. \quad (11)$$

The integral $S(\epsilon, k)$ can be expressed in closed form by means of the simple analytic expressions for the total cross section given by Haug (1981) (see Appendix A), so that the computation of the rate (11) for γ–e pair production requires only a single numerical integration.

This method benefits from the fact that for $k > 15$ an analytical approximation to dN_e/dt can be derived which yields accurate values for temperatures $T < 2 \times 10^9$ K (see Eq. (A.8)).

2.2. Photon pair production on nuclei

For the temperatures considered ($\Theta < 200$ or $T < 1.2 \times 10^{12}$ K) the nuclei in the plasma have velocities small compared to the speed of light. Hence the production of pairs can be calculated in the nucleus rest system with the threshold $2mc^2 \approx 1$ MeV. Then the production rate is given independently of Θ by

$$\frac{dN_Z(k)}{dt} = c \sum_Z n_Z \sigma_Z(k) \quad (12)$$

where n_Z is the number density of ions with atomic number Z. In Born approximation[1] the cross section for pair production in the Coulomb field of nuclei of charge Ze is Z^2 times $\sigma_p(k)$, the cross section in the field of protons (Jauch & Rohrlich 1976). Introducing the elemental abundances relative to hydrogen, n_Z/n_H, Eq. (12) can be written as

$$\frac{dN_Z(k)}{dt} = c n_H \left[1 + \sum_{Z\ge 2} Z^2 (n_Z/n_H)\right] \sigma_p(k). \quad (13)$$

Using the abundances compiled by Anders & Grevesse (1989), the sum has the value 0.36 yielding

$$\frac{dN_Z(k)}{dt} \approx 1.36\, c n_H \sigma_p(k). \quad (14)$$

Maximon (1968) has derived two simple series expansions of the total cross section for pair production in the Coulomb field of a nucleus. With the aid of these expressions the production rate (14) can be easily evaluated.

[1] Since hydrogen and helium are the most abundant elements, the Born approximation, valid for small values of Z, is a good approximation.

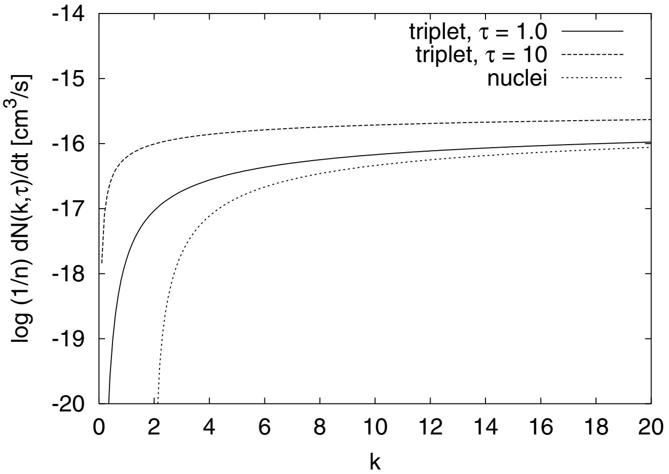

Fig. 1. Photon pair production rate by the triplet process in plasmas of temperature $\Theta = 1$ and $\Theta = 10$ in comparison with the production rate on nuclei.

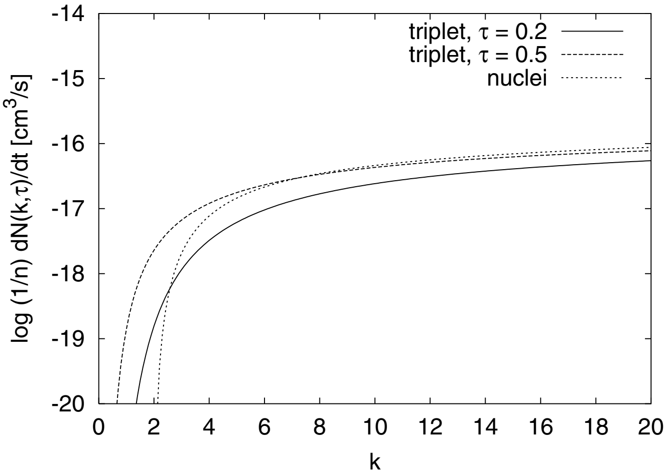

Fig. 2. Photon pair production rate by the triplet process in plasmas of temperature $\Theta = 0.2$ and $\Theta = 0.5$ in comparison with the production rate on nuclei.

3. Results

Figure 1 shows the pair production rate (11) by the triplet process in hot plasmas with $\Theta = 1$ and $\Theta = 10$ compared with the rate (14) of pair production on nuclei. At very high temperatures the triplet production rate is predominant, in particular at photon energies $k < 10$. The pair production on nuclei decreases steeply near the threshold of $h\nu \approx 1$ MeV. At these low photon energies the triplet production is enhanced by the kinetic energy of the thermal electrons and positrons. At lower temperatures of the plasma this effect is still existing even if it is less pronounced, as depicted in Fig. 2. Here the plasma temperatures are $\Theta = 0.2$ and $\Theta = 0.5$, corresponding to $T = 1.2 \times 10^9$ K and 3.0×10^9 K, respectively. Still the triplet production rate exceeds the production rate $\mathrm{d}N_Z/\mathrm{d}t$ for $h\nu < 1$ MeV and must not be neglected even at high photon energies. Modeling of hard X-ray spectra from stellar black-hole binaries and of Seyfert galaxies results in $k_B T = 50$ to 150 keV or $\Theta = 0.1$ to 0.3 (Zdziarski et al. 2000), which is comparable with the lower temperature of Fig. 2.

Acknowledgements. The author is grateful to the referee for constructive comments which led to major improvement of the work.

Appendix A: Calculation of the function $S(\epsilon, k)$ and the pair production rate $\mathrm{d}N_e/\mathrm{d}t$

Using the approximate formulae of Haug (1981) for the total cross section of triplet production, the function $S(\epsilon, k)$ (Eq. (10)) can be calculatedanalytically. In the four different ranges of κ it takes the form:

a) $4 \le \kappa \le 4.6$

$$S_1(\kappa) \approx \Big[0.925 \times 10^{-3}\kappa^6 - (0.1812/7)\kappa^5 + (1.8371/6)\kappa^4 - 1.97064\kappa^3 + 7.3586\kappa^2 - 15.4496\kappa + 14.9952\Big]\kappa^2; \tag{A.1}$$

b) $4.6 \le \kappa \le 6$

$$S_2(\kappa) \approx \Big[0.291407 - (0.29842/3)\kappa + 0.010885\kappa^2 - 2.5954 \times 10^{-4}\kappa^3\Big]\kappa^2; \tag{A.2}$$

c) $6 \le \kappa \le 18$

$$S_3(\kappa) \approx 4.379308\kappa^2 - 324.3065\kappa + 4391.8925 \ln\left(\frac{\kappa + 25.511080}{\kappa + 2.3496166}\right) + 4348.85016 \ln\left(1 + 0.4648\kappa + 0.016683\kappa^2\right); \tag{A.3}$$

d) $\kappa \ge 18$

$$S_4(\kappa) \approx \kappa^2\left[\frac{14}{9}\ln(2\kappa) - \frac{130}{27}\right] + \kappa\left[54.626 - 26.726\ln(2\kappa) + 7.863\ln^2(2\kappa) - \frac{4}{3}\ln^3(2\kappa)\right]. \tag{A.4}$$

In Eqs. (A.1) to (A.3) it is important to retain the digits of the constants because the definite integral (10) is the small difference of two large quantities. If $\kappa_0 = 4$ and the upper limit of the integral (10) is $\kappa_1 \ge 18$, the approximations (A.1) to (A.4) result in

$$\int_4^{\kappa_1} \kappa \frac{\sigma_t(\kappa)}{\alpha r_0^2} \approx \kappa_1\left\{-\frac{4}{3}\ln^3(2\kappa_1) + 7.863\ln^2(2\kappa_1) + \left(\frac{14}{9}\kappa_1 - 26.726\right)\ln(2\kappa_1) - \frac{130}{27}\kappa_1 + 54.626\right\} - 66.60984751. \tag{A.5}$$

For $k > 15$ it is possible to calculate the pair production rate $\mathrm{d}N_e/\mathrm{d}t$ in closed form. Assuming that the lower integration limit $\kappa_0 = k(\epsilon - p)$ of the integral (10) is sufficiently large so that the approximation for large values of κ, Eq. (A.4), can be used, the function $S(\epsilon, k)$ is given by

$$\begin{aligned} S(\epsilon,k) &= \int_{k(\epsilon-p)}^{k(\epsilon+p)} \kappa\,\frac{\sigma_t(\kappa)}{\alpha r_0^2}\,\mathrm{d}\kappa \\ &\approx k^2\left\{\frac{28}{9}\left[(2\epsilon^2-1)\ln(\epsilon+p)+2\epsilon p\ln(2k)\right]-\frac{520}{27}\epsilon p\right\} \\ &\quad -k\left\{\frac{8}{3}\epsilon\ln^3(\epsilon+p)+8p\ln^2(\epsilon+p)\ln(2k)\right. \\ &\quad +8\epsilon\ln(\epsilon+p)\ln^2(2k)+\frac{8}{3}p\ln^3(2k) \\ &\quad -15.726[p\ln^2(\epsilon+p)+2\epsilon\ln(\epsilon+p)\ln(2k) \\ &\quad +p\ln^2(2k)] \\ &\quad \left.+53.452[\epsilon\ln(\epsilon+p)+p\ln(2k)]-109.252\,p\right\}. \end{aligned} \tag{A.6}$$

The integration over ϵ in Eq. (11) can now be performed partly exactly in terms of modified Bessel functions $K_n(1/\Theta)$ and partly by expanding the logarithms $\ln^n(\epsilon+p)$ into powers of p. The result can be written as

$$\begin{aligned} \frac{\mathrm{d}N_e(k,\Theta)}{\mathrm{d}t} &\approx \alpha r_0^2 c n_e\left\{\frac{28}{9}\left[\ln(2k)+2\Theta(1-2\Theta^2)\frac{K_1(1/\Theta)}{K_2(1/\Theta)}\right.\right. \\ &\quad \left.+2\Theta^2\right]-\frac{218}{27}-\frac{\Theta}{k}\left[4\ln^2(2k)-\left(3.726+\frac{16}{3}\Theta^2\right.\right. \\ &\quad \left.+\frac{64}{5}\Theta^4+\frac{3072}{35}\Theta^6\right)\ln(2k)+3.137+6\Theta \\ &\quad +10.484\Theta^2+\frac{107}{12}\Theta^3+25.1616\Theta^4 \\ &\quad \left.+37.0281\Theta^5+172.5367\Theta^6\right] \\ &\quad -\frac{1}{k}K_1(1/\Theta)/K_2(1/\Theta)\left[\frac{4}{3}\ln^3(2k)\right. \\ &\quad -(3.863+8\Theta^2)\ln^2(2k)+(11.0+11.452\Theta^2 \\ &\quad +16\Theta^4+51.2\Theta^6)\ln(2k)-27.9+4\Theta \\ &\quad -14.137\Theta^2-\frac{101}{6}\Theta^3-31.452\Theta^4 \\ &\quad \left.\left.-30.7896\Theta^5-100.6464\Theta^6-164.4831\Theta^7\right]\right\}. \end{aligned} \tag{A.7}$$

Expanding also the modified Bessel functions $K_n(1/\Theta)$, this yields, up to order Θ^7,

$$\begin{aligned} \frac{\mathrm{d}N_e(k,\Theta)}{\mathrm{d}t} &\approx \alpha r_0^2 c n_e\left\{\frac{28}{9}\left[\ln(2k)+2\Theta-\Theta^2-\frac{1}{4}\Theta^3+\frac{9}{4}\Theta^4\right.\right. \\ &\quad \left.-\frac{345}{64}\Theta^5+\frac{165}{16}\Theta^6-\frac{9585}{512}\Theta^7\right]-\frac{218}{27} \\ &\quad -\frac{1}{k}\left[\left\{\frac{4}{3}-2\Theta+\frac{5}{2}(\Theta^2-\Theta^3)+\frac{45}{32}\Theta^4+\frac{15}{8}\Theta^5\right.\right. \\ &\quad \left.-\frac{2475}{256}\Theta^6+\frac{225}{8}\Theta^7\right\}\ln^3(2k) \\ &\quad -\Big(3.863-9.7945\Theta+15.2431\Theta^2-19.2431\Theta^3 \\ &\quad +19.07426\Theta^4-9.5677\Theta^5 \\ &\quad -19.573\Theta^6+92.735\Theta^7\Big)\ln^2(2k) \\ &\quad +\Big(11.0-20.226\Theta+32.077\Theta^2-43.1363\Theta^3 \\ &\quad +49.0741\Theta^4-42.80375\Theta^5+13.5175\Theta^6 \\ &\quad +53.5642\Theta^7\Big)\ln(2k)-27.9+48.987\Theta \\ &\quad -66.4495\Theta^2+74.6687\Theta^3-60.718\Theta^4 \\ &\quad \left.\left.+1.4788\Theta^5+148.173\Theta^6-494.888\Theta^7\right]\right\}. \end{aligned} \tag{A.8}$$

Equation (A.8) is an excellent approximation for $T < 2\times10^9$ K ($\Theta < 0.34$) and $k > 15$. The corresponding formula up to order Θ^2 is still very accurate except for photon energies less than $k = 25$.

References

Anders, E., & Grevesse, N. 1989, GeCoA, 53, 197

Bisnovatyi-Kogan, G. S., Zel'dovich, Y. B., & Sunyaev, R. A. 1971, Sov. Astron., 15, 17

Blandford, R. D., & McKee, C. F. 1976, PhFl, 19, 1130

Bonometto, S. A., Lucchin, F., & Marcolungo, P. 1974, A&A, 31, 41

Dermer, C. D., & Schlickeiser, R. 1991, A&A, 252, 414

Haug, E. 1975, Z. Naturforsch. 30a, 1099

Haug, E. 1981, Z. Naturforsch. 36a, 413

Jauch, J. M., & Rohrlich, F. 1976, The Theory of Photons and Electrons, 2nd edition (Berlin: Springer-Verlag)

Johnson, W. N., McNaron-Brown, K., Kurfess, J. D., et al. 1997, ApJ, 482, 173

Lightman, A. P. 1982, ApJ, 253, 842

Mastichiadis, A., Marscher, A. P., & Brecher, K. 1986, ApJ, 300, 178

Mastichiadis, A. 1991, MNRAS, 253, 235

Mastichiadis, A., Protheroe, R. J., & Szabo, A. P. 1994, MNRAS, 266, 910

Maximon, L. C. 1968, J. Res. NBS 72B, 79

McConnell, M. L. M., Zdziarski, A. A., Bennett, K., et al. 2002, ApJ, 572, 984

Svensson, R. 1982, ApJ, 258, 335

Svensson, R. 1984, MNRAS, 209, 175

Weaver, T. A. 1976, Phys. Rev. A, 13, 1563

Zdziarski, A. A. 1985, ApJ, 289, 514

Zdziarski, A. A., Poutanen, J., & Johnson, W. N. 2000, ApJ, 542, 703