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Rotating central objects with hard surfaces: A pseudo-Newtonian potential for relativistic accretion disks

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Abstract. Here we furnish a pseudo-Newtonian potential for accretion disk modeling around rotating central objects having a hard surface. Thus, the potential may be useful to describe the accretion disk around rotating neutron stars and strange stars. The potential can describe the general relativistic effects in accretion disks properly, essentially studying the relativistic hydrodynamics of disks. To understand the fluid dynamics of the accretion disk around rotating central objects with a hard surface, it is necessary to incorporate the rotation of the central object, which affects the inner edge of the disk. We derive our potential, starting from the Hartle Thorne metric. The potential can reproduce all the essential properties of general relativity with a 10% error at most, for slowly rotating neutron/strange stars.

Key words. accretion, accretion disks – gravitation – relativity

1. Introduction

It is an established fact in astronomy and astrophysics that one can study the accretion disk phenomenon by using a pseudo-Newtonian method. This is favoured by physicists so as to avoid complex and cumbersome general relativistic equations and is effective in mimicing the geometry of space-time. This model was first stated by Shakura & Sunyaev (1973) when they proposed the simple Newtonian potential for non-rotating black holes. As the relativistic effects are extremely important near black holes, this potential is unable to describe the essential inner properties of the disk.

Paczyński & Wiita (1980) modified this Newtonian potential in conformity with the Schwarzchild geometry, which can naturally reveal almost all the properties of the disk around non-rotating black holes, even those of the innermost part of the disk. The potential can reproduce the marginally stable orbit (r_s), marginally bound orbit (r_b) and efficiency per unit mass at the last stable circular orbit (E_s). The last parameter agrees within a 10% error but the first two cases exactly match with that of the metric. Not only can the relativistic fluid dynamics of the accretion disk be studied using this pseudo-potential, other properties of the disk like spectral studies can also be determined around non-rotating black holes. After a gap of ten years, Nowak & Wagoner (1991) proposed another potential for the accretion disk around non-rotating black holes. This potential also can mimic most of the properties of the disk governed by Schwarzschild geometry. However, the epicyclic frequency of the disk can be best analyzed with that potential.

Artemova et al. (1996) proposed pseudo-potentials that can describe the accretion disk around rotating black holes. These potentials were well analyzed by a number of astrophysicists. Very recently, Mukhopadhyay (2002a) and Mukhopadhyay & Misra (2003) presented new potentials, using which, properties of the accretion disk can be very well described in Kerr geometry. A more interesting methodology was adopted by Mukhopadhyay (2002a) to describe the accretion disk around rotating black holes that can be used to derive the pseudo-potential for any metric according to the physics concerned. The potentials proposed by Mukhopadhyay & Misra (2003) can be used for the time-dependent simulation of an accretion disk around rotating black holes and neutron stars.

All the abovementioned potentials are meant for accretion disk around black holes. However, herein, we derive a pseudo-potential for an accretion disk around rotating central objects with a hard surface, mainly neutron stars, strange stars, white dwarfs and other high gravity stars. As the radius of a white dwarf is very large, general relativity is not of much importance in the accretion disk around it. The basic differences between the accretion disk having a centrally hard surface and that of horizon are (Mukhopadhyay 2002b) its (1) inner boundary condition, (2) metric, (3) formation of the shock and (4) luminosity of the disk.

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In the case of a hard surface, inflowing matter has to be stopped at the extreme inner edge of the disk as it represents an extended object whose space-time metric is described by Hartle-Thorne (1968). However, for the non rotating case the metric reduces to the Schwarzschild geometry. Hence, it is quite obvious that the geometry of space time remains the same for central objects having a hard surface and event horizon, when they are non-rotating. Herein, we will call the class of central objects having a hard surface GOHS to distinguish it from a black hole. Around a black hole, only one stable shock formation is possible. In the case of an accretion disk around GOHS, the formation of the shock in a particular matter flow (up to the stellar surface) may happen twice in certain situations (Mukhopadhyay 2002b). As far as the luminosity is concerned, it is greater in the case of an accretion disk around GOHS than that of a black hole, which mostly results from the difference in the nucleosynthesis between them.

Though various pseudo-potentials are known to describe the various accretion disk properties of central objects with an event horizon (rotating or non-rotating black holes), which can approximately mimic the general-relativistic space-times, to date no such potential that can describe relativistic fluid dynamical properties of the accretion disk around rotating GOHS are known to the author. Prasanna & Mukhopadhyay (200) studied the general set of fluid equations with the inclusion of the Coriolis force on accretion flows around rotating compact objects. Other work has been caried out in this field, sometimes using the Paczyński & Wiita (1980) potential and by numerical simulation. Here, we will study the relativistic hydrodynamics of the accretion disk around GOHS, with the inclusion of a proper pseudo-Newtonian potential.

It was known from various observational works that cosmic objects are spinning. Iwasawa et al. (1996) showed in their observation of variable iron K emission lines in MCG-6-30-15 that it comes from the inner part of the accretion disk and is strongly related to the rotation of the black hole. As the inner region of the accretion disk is very much influenced by the rotation of the central object, it should be incorporated into theoretical studies. From another observational point of view (Karas & Kraus 1996; Iwasawa et al. 1996), it was found that central black holes in galactic nuclei are rapidly rotating. We see pulsars that are rapidly rotating neutron stars. It was established that the tidal excitations in coalescing binaries involve rapidly exciting neutron stars. Gravity wave oscillations from neutron stars strengthen the fact that neutron stars are rapidly spinning (Fragile et al. 2001). The occurrence of brightness oscillations during thermonuclear X-ray bursts from neutron star low-mass X-ray binaries synchronised with the rotation period of these neutron stars enhances the possibility that these are spinning fast enough to become unstable to gravity wave radiation.

Thus compact objects and other cosmic objects seem to be rotating. As it has been mentioned above that no proper peseudo-potential has yet been established to describe the accretion flows around rotating neutron stars or in general for any GOHS, here we attempt to describe a pseudo-potential that can mimic the geometry of the space-time of rotating hard surfaces and can be further used to study the global solutions of their accretion disks. In the work of Mukhopadhyay (2002a) the pseudo-potential was derived for rotating black holes starting from the Kerr metric. Most of the other potentials for the rotating black holes have no connection with the space-time metric. Here, we follow the same method as Mukhopadhyay (2002a) to derive the potential for the accretion disk around GOHS, starting from the Hartle-Thorne (HT) metric (Hartle & Thorne 1968). As the HT metric is restricted to slow rotations only, our result will be valid only for the cases of slow rotation. As the metric is involved directly in our calculation, it has been easy to reproduce most of the features of Hartle-Thorne geometry by our potential. The potential reduces to that of Paczyński-Wiita (1980) (hereinafter Paczyński-Wiita potential) for the non rotating case.

As these methods (pseudo-Newtonion potential) are approximate, it can be that values of $r_{\rm b}$, $r_{\rm s}$ and $E_{\rm s}$ for both HT and Kerr geometry may be similar at different values of j, which may encourage use of the pseudo-potential described in Mukhopadhyay (2002a), meant for Kerr geometry, to describe the accretion disk around a rotating hard surface. It should be kept in mind that Kerr geometry cannot describe the solution in the interior, as it is meant for the event horizon. On the other hand, HT geometry can describe it. As we are considering here the rotating hard surface (and not the event horizon), for the continuity of the solution from disk to inside the star, the pseudo-Newtonian potential that can mimic the HT metric has to be taken, since the potential corresponding to the Kerr metric by Mukhopadhyay (2002a) is valid only up to the event horizon. In the next section we describe the derivation of our potential. In Sect. 3, we will compare results of the HT geometry with that of our potential. In Sect. 4, we will make our conclusions.

2. Basic equations pseudo-potential

The Lagrangian density for a particle in the Hartle Thorne (neglecting the higher quadrapole terms) (Hartle & Thorne 1968) space-time at the equitorial plane ($\theta = \pi/2$) can be written as:

$$2\mathcal{L} = -\left(1 - \frac{2M}{r} - \frac{2j^2}{r^4}\right)\dot{t}^2 + \left(1 - \frac{2M}{r} + \frac{2j^2}{r^4}\right)^{-1} \dot{r}^2 + r^2\dot{\phi}^2 - \frac{4j}{r}\dot{\phi}\dot{t} \quad (1)$$

where over-dots denote the derivative with respect to the proper-time τ and j denotes the angular momentum of the central object. As the metric is valid only for slowly rotating stars, the rotation parameter j is restricted, and may be at say, $j \leq 0.5$. The geodesic equations of motion are:

$$E = \text{constant} = \left(1 - \frac{2M}{r} - \frac{2j^2}{r^4}\right)\dot{t} + \frac{2j}{r}\dot{\phi}, \quad (2)$$

$$L = \text{constant} = r^2\dot{\phi} - \frac{2j}{r}\dot{t}. \quad (3)$$

For the particle with non-zero rest mass, $g_{\mu\nu}p^\mu p^\nu = -m^2$ (where p^μ is the momentum of the particles and $g_{\mu\nu}$ is the metric). Replacing the solution for $\dot{t}$ and $\dot{\phi}$ from (2) and (3) into (1), we get a differential equation for r,

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 = \left(\frac{4hj^2}{gr^6} + \frac{16j^4}{gr^{10}} - \frac{g}{r^2}\right)L^2 + \left(\frac{h}{g} + \frac{4j^2}{gr^4}\right)E^2 + \left(-\frac{4hj}{gr^3} - \frac{16j^3}{gr^7}\right)EL - gm^2 = \Psi, \quad (4)$$

where $g = \left(1 - \frac{2m}{r} + \frac{2j^2}{r^4}\right)$, $h = \left(1 - \frac{2m}{r} - \frac{2j^2}{r^4}\right)$. Here Ψ can be identified as an effective potential for the radial geodesic motion. The conditions for circular orbits are:

$$\Psi = 0, \qquad \frac{\mathrm{d}\Psi}{\mathrm{d}r} = 0. \quad (5)$$

Solving for E and L from (5), we get:

$$E = \frac{1}{m^2r^3(-4j^2 + Mr^3)}\Bigg[\left(2j^3m^2 + jm^2(4M - 3r)r^3 - \sqrt{m^4(5j^2 + Mr^3)(2j^2 + r^3(-2M + r))^2}\right)$$
$$\times\Bigg\{\left(-12j^4m^2r^2 + 2j^2m^2(9M - 7r)r^5 + m^2M(3M - r)r^8 + 6jr^2\sqrt{m^4(5j^2 + Mr^3)(2j^2 + r^3(-2M + r))^2}\right)/$$
$$\left(36j^4 - r^6(-3M + r)^2 + 12j^2r^3(-3M + 2r)\right)\Bigg\}^{1/2}\Bigg], \quad (6)$$

$$L = \frac{1}{m^2r^3(-4j^2 + Mr^3)}\Bigg[\left(2j^3m^2 + jm^2(4M - 3r)r^3 - \sqrt{m^4(5j^2 + Mr^3)(2j^2 + r^3(-2M + r))^2}\right)$$
$$\times\Bigg\{\left(-12j^4m^2r^2 + 2j^2m^2(9M - 7r)r^5 + m^2M(3M - r)r^8 + 6jr^2\sqrt{m^4(5j^2 + Mr^3)(2j^2 + r^3(-2M + r))^2}\right)/$$
$$\left(36j^4 - r^6(-3M + r)^2 + 12j^2r^3(-3M + 2r)\right)\Bigg\}^{1/2}\Bigg]$$
$$\times\left[\sqrt{\frac{m^4r^7(-4j^2 + Mr^3)^2}{\left[-2j^3m^2 + jm^2r^3(-4m + 3r) + \sqrt{m^4(5j^2 + Mr^3)(2j^2 + r^3(-2M + r))^2}\right]^2}}\right]. \quad (7)$$

Now as standard practice, we can define the Keplerian angular momentum distribution $\lambda_K = \frac{L}{E}$. Therefore, the corresponding centrifugal force in Hartle Thorne geometry can be written as

$$\frac{\lambda_K^2}{r^3} = \frac{m^4r^4(-4j^2 + Mr^3)^2}{\left[-2j^3m^2 + jm^2r^3(-4m + 3r) + \sqrt{m^4(5j^2 + Mr^3)(2j^2 + r^3(-2M + r))^2}\right]^2} = F_r. \quad (8)$$

Thus from above, F_r can be identified as the gravitational force of a rotating central object having a hard surface at the Keplerian orbit in an equatorial plane. The above expression reduces to the Paczyński-Wiita form for $j = 0$, $m = 1$ and $M = 1$. Thus we propose that Eq. (8) is the most general form of the gravitational force corresponding to the pseudo-potential in an accretion disk around a rotating central object having a hard surface in the equatorial plane. It needs to be emphasized that although our theoretical result is valid for slowly rotating GOHS (the HT metric is valid for slowly rotating stars), it has been found observationally that even fast rotating neutron/strange stars have j much less than 0.5. Hence, our result will be able to explain the accretion disk properties for all observed rotating GOHS. Neverthless, theoretically j can be much higher and for any theoretical calculation where $j > 0.5$, our result will not be valid.

3. Comparison of the results for Hartle-Thorne geometry and pseudo-potential

Following Mukhopadhyay (2002a), we will check how correctly the potential ($V_r = \int F_r \mathrm{d}r$) reproduces the values of r_b, r_s and E_s for that of the Hartle-Thorne geometry, which can be established as a good pseudo-potential in accretion disks around any GOHS. The corresponding equations to calculate r_b, r_s and E_s are given in Mukhopadhyay (2002a). In Tables 1−3 we list the corresponding r_b, r_s and E_s at various values of j.

Table 1. Values of r_b.

j	0	0.1	0.2	0.3	0.4	0.5
V_r	4	3.7862	3.5619	3.3255	3.0762	2.8135
HT	4	3.7954	3.5810	3.3557	3.118	2.8689
j	0	−0.1	−0.2	−0.3	−0.4	−0.5
V_r	4	4.2042	4.4	4.5885	4.77	4.9462
HT	4	4.1957	4.3838	4.5684	4.7397	4.9089

Table 2. Values of r_s.

j	0	0.1	0.2	0.3	0.4	0.5
V_r and HT	6.0	5.6648	5.3107	4.9339	4.5299	4.0934
j	0	−0.1	−0.2	−0.3	−0.4	−0.5
V_r and HT	6.0	6.3189	6.6240	6.9170	7.1994	7.4724

Table 3. Values of E_s.

j	0	0.1	0.2	0.3	0.4	0.5
V_r	−0.0625	−0.0664	−0.0711	−0.0769	−0.0843	−0.0943
HT	−0.0572	−0.0607	−0.0649	−0.0701	−0.0767	−0.0857
j	0	−0.1	−0.2	−0.3	−0.4	−0.5
V_r	−0.0625	−0.0592	−0.0563	−0.0538	−0.0516	−0.0497
HT	−0.0572	−0.0542	−0.0517	−0.0494	−0.0474	−0.0457

From Table 1, it is clear that for all values of j, V_r can reproduce the value of r_b in very good agreement with general relativistic (HT) results. The maximum error in r_b is ∼4%. Table 2 indicates that V_r reproduces r_s with 100% accuracy in the Hartle Thorne geometry. Table 3 shows that the maximum possible error in E_s is ∼10%. Thus, the potential V_r will produce a slightly higher luminosity than the general relativistic one in the accretion disk for a particular value of j. Note that for counter-rotating central objects, i.e., retrograde solutions, the errors are less than those for co-rotating ones. Our potential can describe almost all the phenomena consistent with that of Hartle Thorne geometry and thus it is a good pseudo-potential that can mimic the general relativistic result for the accretion disk around any GOHS, particularly close to the equitorial plane.

4. Conclusions

In this paper, we have presented a pseudo-potential to model the accretion disk around rotating central objects with a hard surface, which can be used to study the fluid dynamics of its disk. For the non-rotating case, that is for $j = 0$, $m = 1$ and for $M = 1$ the potential reduces to the well-known Paczyński-Wiita potential. The author does not know of any pseudo-potential derived exclusively for rotating hard surfaces, which can mimic its corresponding general relativistic metric because the studies done using the Newtonian potential, Paczynski-Wiita potential or other analytical potentials have been meant for non-rotating or rotating black holes and not for rotating neutron stars. Following a similar procedure to that of Mukhopadhyay (2002a), the potential has been described. As the potential is derived from the metric itself, its accuracy is much higher. Our potential is valid for both co-rotating and counter-rotating central objects having a hard surface. Counter-rotating results agree better with the general relativistic ones, presumably because of the larger values of r_b and r_s than in co-rotating cases. This potential can be used to study properties of the accretion disks around any rotating central objects having a hard surface, where the general relavistic effect is much stronger. Though this potential is derived from the corresponding metric itself, this is more applicable to study the relativistic fluid dynamics of the accretion disk around rotating hard surfaces. Other properties of the accretion disk, such as spectral studies or epicyclic frequencies may not be analyzed efficiently. If the description of the disk properties is acceptable within 10% accuracy, our potential should be useful. It must however be noticed that as $\theta = \pi/2$ has been chosen at the very beginning of our calculation, the potential is mainly valid at the equatorial plane. These few hindrances notwithstanding, this potential is one of the first to describe the accretion disk properties around any rotating GOHS in the equitorial plane.

We have discussed why we prefer the Hartle-Thorne metric over the Kerr metric in the context of these approximate methods (pseudo-Newtonian potential). Generalisation of the potential describing all properties of the accretion disk around rotating central objects with a hard surface will be attempted in the future. Next, one should study how the rotation of central objects having a hard surface affects the fluid dynamical properties in an accretion disk. How does it affect the parameter region of a disk? Thus, we should study the complete set of global solutions of viscous transonic flows around rotating hard surfaces.

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