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Nonradial oscillations in classical Cepheids: the problem revisited

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ABSTRACT

Context. We analyse the presence of nonradial oscillations in Cepheids, a problem that has not been theoretically revised since the work of Dziembowski (1977, Acta Astron., 27, 95) and Osaki (1977, PASJ, 29, 235). Our analysis is motivated by a work of Moskalik et al. (2004, ASPC, 310, 498), which reports the detection of low-amplitude periodicities in a few Cepheids of the large Magellanic cloud. These newly discovered periodicities were interpreted as nonradial modes.
Aims. Based on linear nonadiabatic stability analysis, our goal is to reanalyse the presence and stability of nonradial modes, taking into account improvement in the main input physics required for the modelling of Cepheids.
Methods. We compare the results obtained from two different numerical methods used to solve the set of differential equations: a matrix method and the Ricatti method.
Results. We show the limitation of the matrix method for finding low-order p-modes ($l < 6$), because of their dual character in evolved stars such as Cepheids. For higher order p-modes, we find excellent agreement between the two methods.
Conclusions. No nonradial instability is found below $l = 5$, whereas many unstable nonradial modes exist for higher orders. We also find that nonradial modes remain unstable, even at hotter effective temperatures than the blue edge of the Cepheid instability strip, where no radial pulsations are expected.

Key words. stars: oscillations

1. Introduction

Cepheids are well-known radial pulsators, oscillating in the fundamental mode or the low overtones. Despite a very abundant literature analysing their radial pulsation properties, theoretically and observationally, only very few studies have been devoted to nonradial oscillations in these stars. Dziembowski (1971) was the first to study nonradial oscillations in Cepheids, based on a quasi-adiabatic linear stability analysis, and found that low-order modes ($l \leq 2$) are stable. Following this study, Osaki (1977) and Dziembowski (1977) showed that higher degree nonradial modes ($l \geq 4$) can become unstable. These analyses highlighted the difficulty of calculating low-order p-modes in such evolved stars. These modes show a dual character as they can also propagate in the gravity-wave region, behaving as high-order gravity modes in the central regions and thus having several thousand nodes there. Since no observational evidence of nonradial modes in Cepheids was available at the time of these studies, no further theoretical analysis of the stability of nonradial modes was performed.

Later on, however, due to advances in spectroscopy and radial velocity measurement techniques, the presence of nonradial modes in classical Cepheids was more convincingly considered to explain anomalous behaviours such as amplitude modulations (Hatzes & Cochran 1995; Van Hoolst & Waelkens 1995; Koen 2001; Kovtyukh et al. 2003). These have been interpreted either as the beating of two linearly unstable radial and nonradial modes with similar frequencies or as a nonlinear interaction between a linearly unstable radial mode and a linearly stable, low degree, low-order nonradial mode

(Van Hoolst & Waelkens 1995; Van Hoolst et al. 1998). The latter hypothesis was theoretically investigated by Van Hoolst & Waelkens (1995), based on frequencies and linear growth rates of the nonradial modes calculated previously by Osaki (1977).

More recently, Moskalik et al. (2004) found low-amplitude secondary periodicities in a few first overtone Cepheids of the Large Magellanic Cloud, with frequencies close to the unstable, radial, first overtone. These newly discovered periodicities were interpreted as nonradial modes. They provide, to our knowledge, the first direct evidence for the presence of nonradial modes in classical Cepheids. But unfortunately, these observations have not been confirmed yet. The observations of Moskalik et al. (2004) are based on photometrical data, implying that if nonradial modes are indeed detected, they must be low-degree modes. According to Osaki (1977), however, these modes should be stable. Motivated by the observations of Moskalik et al. (2004) and the fact that, since the work of Osaki (1977), no updated stability analysis of nonradial modes has been performed, we investigate this problem again, taking into account the improvement in the main input physics (opacities, equation of state) required for modelling Cepheids.

We used two different numerical methods for solving the set of linear pulsation equations. The first one is based on a Henyey-type relaxation scheme, which is the one most commonly used in the community; the second one is based on the Riccati method (see Gautschy & Glatzel 1990, and references therein). The two methods are briefly described in Sects. 2 and 3. Because of the above-mentioned dual character of low-degree p-modes in highly evolved stars, the first method requires an extremely large number of grid-points for the Cepheid model

because of all the nodes in the central region. This method thus suffers from resolution problems and a lack of accuracy. One of our purposes is to analyse and highlight these limitations, which is done in Sect. 2. To overcome the problems, several ideas have been suggested, such as asymptotic methods providing analytical solutions (Dziembowski 1977; Lee 1985). Osaki (1977) found that the stellar envelope could be regarded as a unique pulsating unit, with an appropriate boundary condition at the bottom of the envelope that should take the enery leakage into the core into account. As a more attractive alternative, the Riccati method as used by Gautschy & Glatzel (1990), avoids the extra-complexity of asymptotic methods and can take the entire stellar structure into account, providing a correct description of the energy leakage into the core without resorting to any approximation.

In Sect. 4 we present the outcome of our stability analysis and compare the results obtained with both numerical methods for low-degree p-modes.

2. Linear stability analysis based on the Henyey-method

The pulsation calculations were performed with a nonradial code originally developed by Lee (1985) and previously used by one of the authors for an extensive study of radial pulsations in classical Cepheids (Alibert et al. 1999). We briefly describe the method in Sect. 2.1. A new method of mode classification is presented in Sect. 2.2.

2.1. Linear pulsation equations

The system of linearised equations describing small amplitude stellar pulsations can be found in Unno et al. (1989). The eigenfunctions $X(r, \theta, \phi, t)$, which are solutions to this system of equations, are expressed in terms of spherical harmonics as $X = x(r)Y_m^l(\theta, \phi)e^{i\sigma t}$, where l indicates the degree of the eigenmode and m its azimuthal number. The eigenfrequency of the mode is $\sigma = \sigma_r + i\sigma_i$, with $P = 2\pi/\sigma_r$ the pulsation period, and σ_i characterising the stability of the eigenmode. Positive values of σ_i indicate stable modes. In the following, all frequencies are given in units of $\sigma_0 = \sqrt{\frac{Gm}{R^3}}$, with R the radius and m the mass of the star. Convection is frozen in, assuming that the perturbation of the convective flux in the linearised energy conservation equation is neglected. Though crude, this approximation can be justified as we are mainly interested in Cepheids close to the blue edge of the instability strip, where convection plays a minor role. As shown in Alibert et al. (1999), frozen-in convection provides a theoretical blue edge in good agreement with observed radial fundamental and first overtone Cepheid pulsators. The boundary conditions are those imposed by the regularity of the eigenfunctions (see Unno et al. 1989). A normalisation condition is added: the radial component of the displacement is set to one at the surface of the star. We adopt, as in Unno et al. (1989), the following variables $y_1 = \frac{\xi_r}{r}$, $y_2 = \frac{1}{gr}\left(\frac{p'}{\rho} + \phi'\right) = \frac{\sigma^2 r}{g}\frac{\xi_h}{r}$, $y_3 = \frac{1}{gr}\phi'$, $y_4 = \frac{1}{g}\frac{d\phi'}{dr}$, $y_5 = \frac{\delta S}{C_p}$, and $y_6 = \frac{\delta L_{rad}}{L_{rad}}$, with ξ_r the radial and ξ_h the horizontal component of the displacement respectively, g the gravitational acceleration, ϕ the gravitational potential, p' and ϕ' the Eulerian perturbations of the pressure and gravitational potential, L_{rad} and δL_{rad} the radiative luminosity and its Lagrangian perturbation, δS the Lagrangian

perturbation of the entropy, and C_p the specific heat at constant pressure. The inner boundary conditions are

$$\frac{c_1\sigma^2}{l}y_1 - y_2 = 0 \tag{1}$$

and

$$ly_3 - y_4 = 0, \tag{2}$$

with $c_1 = \frac{m}{m_r}\frac{r^3}{R^3}$, and m_r is the mass inside the sphere of radius r. The outer boundary conditions at $r = R$ are:

$$(l + 1)y_3 + y_4 = 0, \tag{3}$$

$$(2 - 4\nabla_{ad}V)y_1 + 4\nabla_{ad}V(y_2 - y_3) + 4y_5 - y_6 = 0, \tag{4}$$

$$y_1 = 1 \tag{5}$$

with ∇_{ad} the adiabatic gradient and $V = -\frac{d\ln P}{d\ln r}$. Finally, the system of linearised equations is numerically solved with a Henyey-type relaxation method. It requires a guess of the eigenfrequency that is derived from the solution to the adiabatic problem.

2.2. Modal classification

The standard modal classification of nonradial modes, based on the determination of the number of nodes N_g in the gravity-wave and N_p in the acoustic-wave zones (see e.g. Unno et al. 1989) usually fails for evolved stars, as underlined by Dziembowski (1971), because of the dual character of the modes. Modes with p-mode character in the envelope do oscillate rapidly, like g-modes, in the central region. This behaviour is illustrated in Fig. 1, which displays the radial displacement of a p-mode of degree $l = 10$ found in a typical Cepheid model. In this case the number of nodes calculated with the present numerical method is inaccurate and meaningless, with $N_p = 117$ and $N_g = 143$. Another method must be adopted to select and classify p-modes, which are the most interesting ones since they can propagate to the surface and are thus potentially detectable.

Also, a huge number of solutions is found, among which some are unphysical and result from numerical noise. For the selection of physical p-modes, we used a criterion based on the modal kinetic energy, E_c, which reads for a layer between radii r and $r + dr$:

$$E_c = \frac{1}{2}\sigma^2\rho r^2|\delta r|^2,$$

with $|\delta r|^2 = |\xi_r|^2 + l(l + 1)|\xi_h|^2$.

If the kinetic energy of an eigenmode reaches its maximum value in the acoustic-wave zone ($\sigma > N, L$, with N and L the Brunt-Väisälä and the Lamb frequencies, respectively), it is selected as a correct solution, which is illustrated in Fig. 2. Each selected eigenmode, for a given l, is then classified as fundamental mode F or overtone (1H, 2H, etc.) with an increasing value of σ_r. This selection method allows us to find p-modes with a high degree, $l \geq 6$, but fails for lower degrees due to the limitation of the numerical method. Although we find numerous solutions for $l < 6$, many of them have very close values of σ_r but very different values of σ_i. Moreover, they all have "pathological" eigenfunctions, so a selection of the correct solution based on the shape of the eigenfunctions or the kinetic energy is not possible.

Since these problems stem from the rapid oscillations of the eigenfunctions close to the centre of the star, one could think
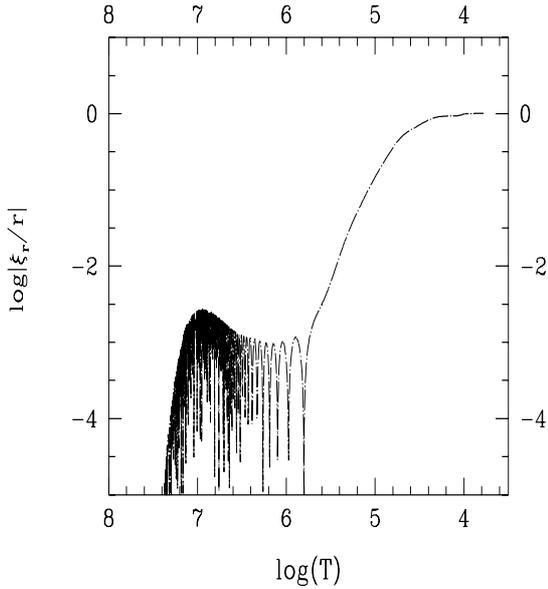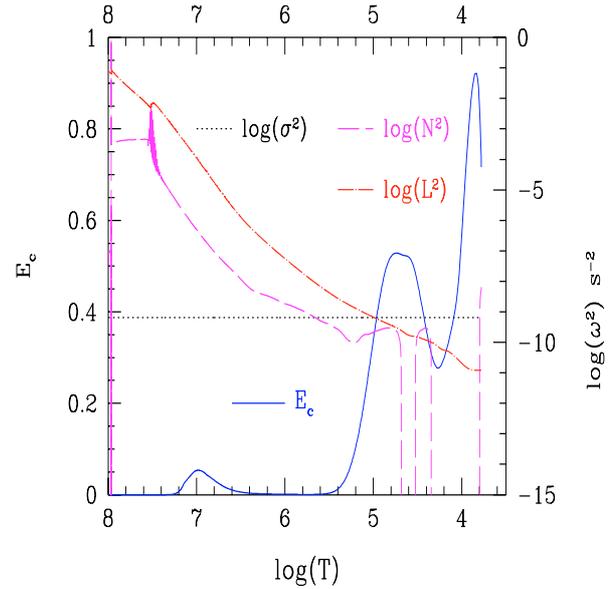
Fig. 1. Radial displacement as a function of temperature of a p-mode with $l = 10$, frequency $\sigma_r = 3.62$ in units of σ_0, for a typical Cepheid model ($m = 5\,M_\odot$, $\log L/L_\odot = 3.1$, $T_{\rm eff} = 5930$ K).

Fig. 2. Kinetic energy E_c (solid line, arbitrary units) for the same eigenmode with $l = 10$ as in Fig. 1. E_c reaches its maximum value in the p-zone, where the eigenfrequency (dotted line) is higher than the Brunt-Väisälä frequency N (dashed line) and higher than the Lamb frequency L (dash-dotted line). All frequencies are in s^{-1} (right-hand y-axis). This eigenmode fullfills the selection criterion of a p-mode (see text Sect. 2.2).

of using the quite common "envelope model" approach, namely by solving the dispersion equation using envelope models rather than complete stellar models. This approach, however, is not useful in the present context since one needs to transfer the unambiguous inner boundary conditions taken at $r = 0$ to the inner edge of the envelope. Since there is no zone where the evanescent oscillation could be rapidly switched off, it is quite difficult to choose the correct inner boundary conditions with envelope models. To get rid of this problem, the inner boundary conditions should be taken deep enough; if not, they will affect the solution. We found that using envelope models requires the same typical resolution as the one required with complete stellar models. There is thus no advantage to using an envelope model rather than a complete one since the size of the matrix that needs to be inverted is approximately the same. If the spatial resolution of envelope models is too low, this implies adopting arbitrary inner boundary conditions, leading to very uncertain results for the values *and* the sign of σ_i. The analysis of non-radial pulsations in Cepheids based on envelope models should thus be used with caution. Finally, note that all results presented in this paper with the Henyey-method are obtained with complete stellar models and a typical number of grid-points of 5000, in agreement with the required resolution estimate based on a WKB approach.

3. The Riccati method

Because of the difficulties encountered with this numerical method, an independent check of the results both for high and low degree p-modes is mandatory. Thereby it will be particularly interesting to check whether the solutions selected with the previously described matrix method are correct and no physical solution has been missed.

One method that does not suffer from resolution problems and provides eigenvalues and eigenfunctions with prescribed accuracy irrespective of the order of an eigenfunction is the Riccati method. Its application to stellar stability problems is described in Gautschy & Glatzel (1990) for radial perturbations and in Glatzel & Gautschy (1992) for nonradial perturbations.

We only briefly summarise the basic essentials of the approach here and refer the reader to these publications for details.

The rapidly oscillating character in the central regions of the eigensolutions considered here implies severe resolution problems due to the limited size of matrices, if the eigenvalue problem is solved on the basis of a matrix eigenvalue problem. This basic difficulty, which always occurs for high-order eigenfunctions, can in principle be overcome by shooting methods, which do not rely on the inversion of large matrices. Moreover, using a shooting method, the local stepsize can be adjusted to match prescribed accuracy requirements. However, for higher-order boundary value problems (the nonadiabatic nonradial stability analysis implies the solution of a sixth order complex differential equation), shooting methods tend to suffer from the parasitic growth problem and are numerically unstable. Moreover, with increasing order the number of parameters that need to be iterated in the shooting process becomes prohibitively large. Therefore shooting methods are usually not considered for the solution of high-order boundary value problems.

One way to overcome the parasitic growth problem and to reduce the parameters to be iterated to the complex eigenvalue, while still preserving the basic advantages of a shooting method, consists in transforming the linear boundary eigenvalue problem to a nonlinear one implying a Riccati type equation. For the shooting process, we thus obtain unambiguous initial conditions at both boundaries, and the only parameter that needs to be iterated is the eigenfrequency. In addition, integration of the nonlinear Riccati equation as an initial value problem is numerically stable. As a result, the integration of the Riccati equation provides a complex (determinant) function of the complex frequency. Zeros of this determinant correspond to the eigenvalues sought.

The latter property of the Riccati method offers an additional advantage: eigensolutions can be determined without resorting to any initial guess. Using matrix methods, these are usually

Table 1. Comparison between the Riccati (Ric.) and the Henyey (Heny.) methods for linear stability analysis (Cepheid model: mass $m = 5 \, M_\odot$, metallicity (in mass fraction) $Z = 0.01$, $\log L/L_\odot = 3.1$, $T_{\mathrm{eff}} = 5930$ K, $\sigma_0 \sim 1.42 \times 10^5$ cgs).

l	mode	σ_r/σ_0 (Ric.)	σ_i/σ_0 (Ric.)	σ_r/σ_0 (Heny.)	σ_i/σ_0 (Heny.)	Period (days)
0	F	2.99	-3.8×10^{-4}	3.02	-4.4×10^{-4}	3.43
	1H	4.23	-6.3×10^{-3}	4.25	-7.5×10^{-3}	2.43
	2H	5.40	-2.7×10^{-3}	5.40	-4.2×10^{-3}	1.92
6	F	3.13	2.0×10^{-2}	3.20	1.8×10^{-2}	3.24
	1H	4.57	-3.9×10^{-3}	4.58	-5.0×10^{-4}	2.26
	2H	5.98	-1.1×10^{-3}	5.96	7.5×10^{-3}	1.74
10	F	3.60	-2.5×10^{-3}	3.61	-1.9×10^{-3}	2.87
	1H	5.23	-9.8×10^{-3}	5.22	-5.2×10^{-3}	1.98
	2H	6.88	2.4×10^{-2}	6.83	3.8×10^{-2}	1.52

obtained on the basis of approximations (e.g., the solution of the adiabatic problem), which may lead to a lack of unexpected solutions. Using the Riccati procedure, the full set of equations without any approximation is always integrated to provide the determinant function. By tabulating the determinant as a function of the complex frequency, a coarse determination of the eigensolutions can be made, which subsequently may be improved by iteration. Due to the existence of the determinant function, there is no problem of spurious eigenvalues when using the Riccati method.
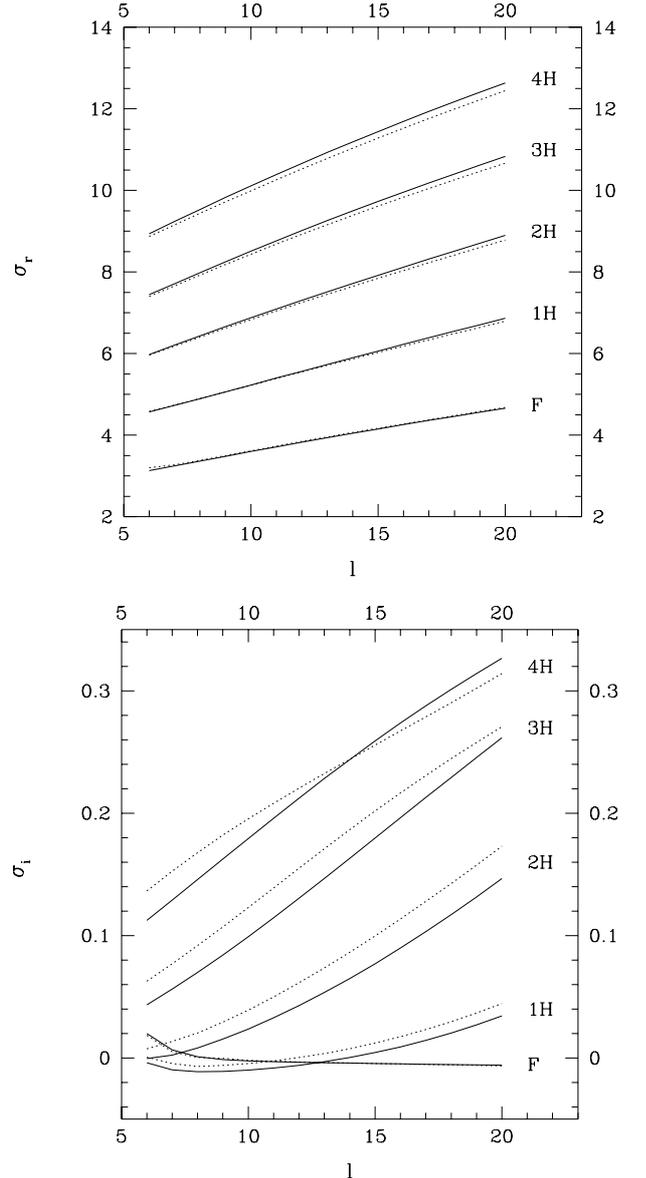
4. Results

4.1. Comparing the two methods

We compared results obtained with the two methods described in Sects. 2 and 3 for a specific Cepheid model. Since our work is motivated by the observations of Moskalik et al. (2004), we selected a model that could describe the properties of one of the observed LMC targets where secondary periodicities were detected. The observed Cepheid, LMC-SC2-208897, is a first overtone pulsator with a period of $P_1 = 2.42$ days and absolute magnitude $M_V \sim -3.2$. Adopting the same input physics and evolutionary code as in Alibert et al. (1999), we found that a model with mass $m = 5 \, M_\odot$, metallicity (in mass fraction) $Z = 0.01$, $\log L/L_\odot = 3.1$, $T_{\mathrm{eff}} = 5930$ K, and close to the end of central He burning (central mass fraction of He $Y_c \sim 4 \times 10^{-2}$) provides a good fit for the observed magnitude and period of LMC-SC2-208897. The calculated first overtone period is unstable with a period $P_1 = 2.43$ days and the absolute magnitude is $M_V = -3$, in excellent agreement with the observed values (see Alibert et al. 1999 for the determination of absolute magnitudes).

Excellent agreement is found between the two methods for the periods and growth rates of radial modes (see Table 1, positive values of σ_i indicate stable modes). Figure 3 compares the values of σ_r and σ_i obtained with both methods, up to $l = 20$. The values of σ_r agree within less than 2%. Differences are found for the values of σ_i for overtones, whereas the agreement is excellent for the fundamental mode (see Fig. 3).

For the above-mentioned effective temperature and luminosity, a variation in the mixing length between one pressure scale height H_P and $2 \times H_P$ or of the metallicity between $Z = 0.01$ and $Z = 0.02$ have only a slight influence: σ_r changes by less than 2.5% and σ_i by less than 8% (if $|\sigma_i| \gtrsim 5 \times 10^{-3}$). Larger differences can appear, however, for very low values of σ_i


Fig. 3. Comparison, for our selected Cepheid model ($m = 5 \, M_\odot$, $\log L/L_\odot = 3.1$, $T_{\mathrm{eff}} = 5930$ K) between the results obtained with the Henyey method (see Sect. 2, dotted lines) and the Riccati method (see Sect. 3, solid lines). The upper panel displays the real part of eigenfrequencies of p-modes as a function of the degree l and the lower panel the imaginary part, in units of σ_0. Each curve is labelled, on the right hand side, according to the modal classification. Note that for the fundamental mode, the curves corresponding to the values of σ_i obtained with both methods are indistinguishable.

($|\sigma_i| \lesssim 5 \times 10^{-3}$), when using the matrix method described in Sect. 2.

We also compared the results obtained with both methods for different effective temperatures, adopting the same mass and luminosity as the above-mentioned Cepheid model. Figure 4 shows the results for the fundamental mode, 1H and 2H with degree $l = 10$. Here again, excellent agreement is found for the values of σ_r, but differences appear for the values of σ_i for the overtones. In general, both methods agree on the sign of σ_i, providing the same results concerning the stability properties of modes, except for very low values of σ_i ($|\sigma_i| \lesssim 5 \times 10^{-3}$).
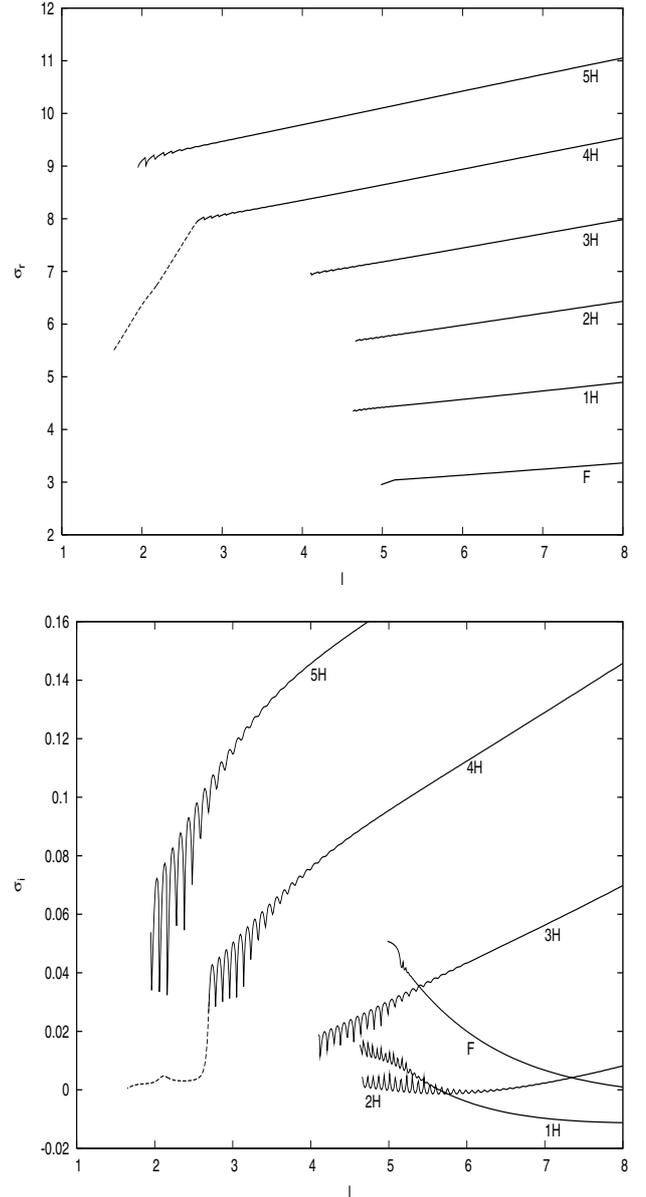
Both for low and high degree modes, we tested the dependence on the outer boundary conditions of the results for the
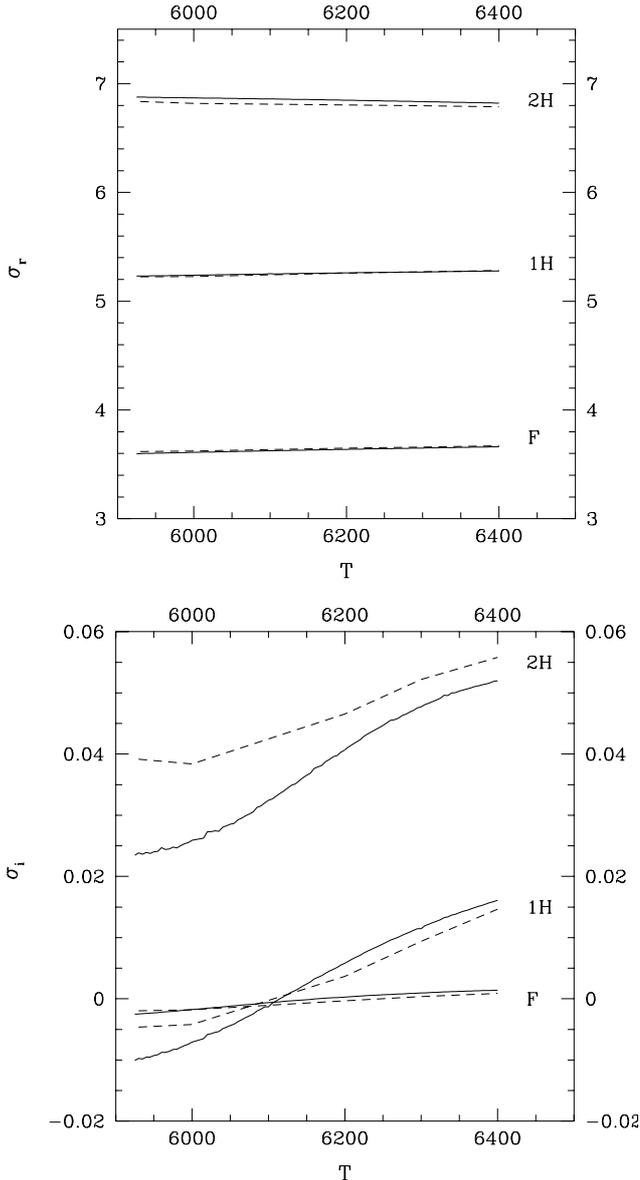
Fig. 4. Variation in σ_r and σ_i, in units of σ_0, as a function of effective temperature for models with $m = 5\ M_\odot$, $\log L/L_\odot = 3.1$, and $Z = 0.01$. Different p-modes with degree $l = 10$ are displayed, as indicated on the figure. The solid line corresponds to the Riccati method and the dashed line to the Henyey method.

Fig. 5. Variation in σ_r and σ_i, in units of σ_0, for the modes indicated as a function of the harmonic degree for low values of l and the Cepheid model having $m = 5\ M_\odot$, $\log L/L_\odot = 3.1$, $T_{\text{eff}} = 5930$ K, and $Z = 0.01$. All results are based on the Riccati method. The bumps for low values of l are due to an interaction of p- and g-modes through avoided crossings. The dashed line indicates the run of a selected g-mode.

stability analysis. This is necessary, since the outer boundary of the stellar model (the photosphere) does not correspond to the physical boundary of the star. Therefore the thermal and mechanical boundary conditions are ambiguous there. In fact, we found a sensitive dependence on boundary conditions even within the physically admissible ones both for radial and non-radial modes. The results shown in the paper, however, are based on the most conservative condition with respect to stability; i.e., we have chosen the condition that provides the least unstable modes. It corresponds to the requirement of vanishing Lagrangian pressure and temperature perturbation. All other boundary conditions, in particular at the inner boundary, are unambiguous.

In contrast to the thermal and mechanical outer boundary conditions, the influence of metallicity on the stability of all the models investigated is rather weak. Relative differences between the results for $Z = 0.01$ and $Z = 0.02$ amount to 10 per cent at maximum for σ_i.

4.2. Results for low-degree p-modes

Modes of degree below $l \approx 6$ are extremely difficult to find with matrix methods. Indeed, only high overtones can be found (e.g. 5H for $l = 2$). Some results obtained in this regime are based on the Riccati method and shown in Fig. 5. Both the frequencies of p- and g-modes decrease with the harmonic degree. However, while p-modes reach finite frequency values for $l \to 0$, the frequencies of g-modes vanish in this limit. The stellar models considered here exhibit a propagation region for gravity waves close to the centre of the star, which for high values of the harmonic degree is detached and shielded from the acoustic

propagation region by an efficient evanescent barrier. It allows for gravity modes with frequencies in the range of p-modes up to quite a high order. Since the decrease in frequencies with l is stronger for the g-modes, this leads to multiple crossings and resonances between p- and g-modes. Due to the efficient evanescent barrier for high values of l, the interaction of p- and g-modes is weak at resonances above $l \approx 6$ and leaves them unaffected. For $l < 6$, however, the barrier becomes weaker and the resonances at the crossing of g- and p-modes imply significant interaction and unfold into avoided crossings. Bumps both in the real (less pronounced) and imaginary parts of the eigenfrequencies as a function of l found in Fig. 5 are parts of these avoided crossings. For illustration a part of the run of a g-mode with l is shown, and l is taken as a real parameter. The latter allows one to follow modes continuously (of course, only integer values are physically meaningful). As a consequence of the resonances, modes with $l < 6$, in particular the so-called p-modes, should not be classified as either p- or g-modes. Physically they instead exhibit properties of both types of modes.

5. Discussion and conclusion

The stellar model whose parameters are thought to match the properties of the Cepheid observed by Moskalik et al. (2004) is found to be unstable both with respect to radial and nonradial perturbations. Following modes up to $l = 500$, we find the following unstable modes: the fundamental p-mode for $l = 0$ and between $9 \leq l \leq 368$, the first overtone for $l = 0$ and between $6 \leq l \leq 13$, and the second overtone for $l = 0, 5,$ and 6. No nonradial instability was found below $l = 5$. Specific studies based on the Ricatti method for $l = 1$ and 2 have not revealed any instability in the p-mode range. Moreover, from extrapolating the curves in Fig. 5 we do not expect instability for $l = 3$ and 4. We can thus conclude that if low-degree p-modes with $l \leq 2$ were necessary to explain the observations of Moskalik et al. (2004), they cannot be attributed to a *linear instability* of these modes. Rather, other effects, such as, e.g., nonlinear mode coupling, have to be invoked for an explanation of their excitation.

With respect to the dependence on effective temperature of the instability of radial and nonradial modes, we find the unstable radial modes (F, 1H, 2H) to stabilise above 6200 K, for our test case luminosity log $L/L_{\odot} = 3.1$. The same is true for nonradial modes with $l \leq 10$. However, fundamental p-modes with $l > 10$ may still be unstable at temperatures above 6200 K, where no radial mode is found to be unstable. For example, the fundamental p-mode for $l = 15$ is unstable up to 6290 K, and the corresponding mode for $l = 60$ is unstable up to 6640 K. We thus conclude that it might be worthwhile to search for nonradial pulsations with $l \geq 5$ in Cepheids, although they may be observationally difficult to detect. Particularly promising are observations of objects with $T_{\text{eff}} > 6200$ K, where no radial pulsations are expected and – according to theory – only nonradial pulsations should prevail. Observations of nonradial pulsations in Cepheids could then provide an interesting and unique tool for sounding their inner structure.

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