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Asteroid occultations today and tomorrow: toward the GAIA era

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ABSTRACT

Context. Observation of star occultations is a powerful tool to determine shapes and sizes of asteroids. This is key information necessary for studying the evolution of the asteroid belt and to calibrate indirect methods of size determination, such as the models used to analyze thermal infrared observations. Up to now, the observation of asteroid occultations is an activity essentially secured by amateur astronomers equipped with small, portable equipments. However, the accuracy of the available ephemeris prevents accurate predictions of the occultation events for objects smaller than ~100 km.
Aims. We investigate current limits in predictability and observability of asteroid occultations, and we study their possible evolution in the future, when high accuracy asteroid orbits and star positions (such as those expected from the mission Gaia of the European Space Agency) will be available.
Methods. We use a simple model for asteroid ephemeris uncertainties and numerical algorithms for estimating the limits imposed by the instruments, assuming realistic CCD performances and asteroid size distribution, to estimate the expected occultation rate under different conditions.
Results. We show that high accuracy ephemerides which will be available in the future will extend toward much smaller asteroids the possibility of observing asteroid occultations, greatly increasing the number of events and objects involved. A complete set of size measurements down to ~10 km main belt asteroids could be obtained in a few years, provided that a small network of ground-based 1m telescopes are devoted to occultation studies.

Key words. astrometry – occultations – minor planets, asteroids – techniques: high angular resolution

1. Introduction

An accurate knowledge of asteroid sizes and size distribution is a fundamental prerequisite for studying the origin and the collisional evolution of these bodies, including the formation and the dispersion of dynamical families. It also has important implications for our understating of the mechanisms that led to the formation of the terrestrial planets of our Solar System. Furthermore, knowledge of the size of an asteroid allows its bulk density to be inferred when the mass of the body can be independently determined, for instance from the orbital period of an asteroid satellite. Information about the density of asteroids is crucial to study their interiors, and to constrain the collisional processes leading to the formation and evolution of the different populations of these minor bodies (e.g. Britt et al. 2002). Moreover, if the size of an asteroid is known, the albedo of the body can be derived using Eq. (1) (e.g. Bowell et al. 1989), where p_V is the geometric visible albedo and H is the absolute magnitude of the asteroid.

$$D(\text{km}) = \frac{1329}{\sqrt{p_V}} \times 10^{-H/5}. \quad (1)$$

Note that H is a quantity that can be easily obtained from measurements of the apparent brightness of the object, for example by means of photometric observations in the visible light. The distribution of albedos and its correlation with the taxonomic types and the orbital elements of asteroids is a key piece of data in understanding their nature and their origin, and can help to constrain the mineralogical composition of their surfaces. Albedos are also crucial for developing and improving reliable and truly debiased asteroid population models (e.g. Tedesco et al. 2005; Stuart & Binzel 2004).

Asteroid flybys of space probes have provided not only the best "direct" size determination, but also a wealth of information on shapes and surface properties for a very limited number of these bodies. However, a statistically significant sample of asteroid sizes can be obtained only by remote observations. Up to now, mainly indirect methods have been used to determine asteroid diameters: the vast majority of asteroid sizes and albedos have been derived from observations obtained in the thermal infrared (~12–100 μm) by the Infrared Astronomical Satellite (IRAS; Tedesco 1992; Tedesco et al. 2002). However, it is known that IRAS diameters of asteroids are not exempt of systematic errors. They were derived by means of the Standard Thermal Model (STM of Lebofsky & Spencer 1989), whose results are dependent on the value of a model parameter called the *beaming factor* and indicated with η. The value of η was calibrated by forcing the STM to give the correct diameters (as derived from occultation observations) of the asteroids 1 Ceres and 2 Pallas (Lebofsky et al. 1986). Several studies have highlighted, though, that a value of η appropriate for large main belt asteroids, might lead to an underestimation of the true size when applied to small asteroids whose surface thermal characteristics are likely to be different from those of larger bodies (e.g. Spencer et al. 1989; Delbó & Harris 2002; Harris & Lagerros 2002).

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Calibration of asteroid radiometric diameters was possible because, in the past half century, the sizes of the largest asteroids were also derived by measuring the duration of star occultations. This approach has traditionally been flawed by the poor accuracy of the prediction of the asteroid shadow path on Earth, thus keeping the success rate of asteroid occultation very low. The availability of precise star astrometry obtained by the mission Hipparcos of the European Space Agency (ESA), has improved the situation in the last ~10 years (Dunham et al. 2002), strongly increasing the number of successfully measured asteroid occultation chords. Electronic equipments (fast video cameras) have also increased the accuracy of occultation timings, previously obtained mainly by visual observation. However, asteroid occultation observations have remained mainly an activity reserved to observers equipped with small, movable telescopes and equipment, ready to accept a rather high rate of "miss".

In this study, after having reviewed the present state of asteroid occultation studies, we will focus on a very likely evolution of this observational approach, thanks to the future availability of asteroid orbits and star positions whose uncertainties will be reduced by orders of magnitude. In this respect, the astrometric ESA mission *Gaia* (Perryman et al. 2001) will represent a major milestone that will completely change the approach to asteroid occultation studies, opening the way to the possibility of obtaining a more complete mapping of asteroid sizes and albedos. In this paper we address this forthcoming improvement, exploring the path that the approach to asteroid occultations will take in the future and investigating the limits that future observation will be able to approach.

Note that indirect methods of asteroid size and albedo determination, such as those based on polarimetry and thermal infrared observations, will greatly benefit from occultation studies performed on a larger sample of asteroids: it will be possible, for example, to calibrate models used to analyze thermal infrared and visual polarimetric measurements on a larger and more accurate sample of calibration asteroid diameters, with object sizes extending down to some tens of kilometers.

Since better performing telescopes will be needed to push size determination toward smaller and thus fainter asteroids, in this paper, we also study the observing rate of asteroid occultations expected for a fixed observing station using a one-meter-class telescope equipped with a cooled CCD camera capable of a high frame rate.

After having assessed the present state of occultation studies (Sect. 1) we analyze the theoretical occultation rate that is expected for different asteroid sizes (Sect. 2). We then study the role of instrument detection capabilities (Sect. 3) and that of ephemeris accuracy (Sect. 4). The implications of high-accuracy ephemeris are then discussed (Sect. 5).

2. The observation of asteroid occultation: where we are now

2.1. Observational campaigns

The first occultation ever recorded is that of 3 Juno, in 1958. The third one (433 Eros), observed in 1975, was also the first to be observed by multiple sites, allowing the determination of different "chords" of the asteroid (Millis & Dunham 1989). Only 17 positive occultation observations were obtained by the end of 1980.

Coordinated campaigns and last-minute small-field astrometry, aimed both at improving the occultation path and at diffusing the relevant predictions among observer networks, have provided a moderate increase in the rate of successfully measured asteroid occultations events. Up to the early 1990s the probability of observing an occultation chord for a given event was discouraging. However, consistent efforts were rewarded with a total of 60 positive events by 1985, growing to 101 in 1990, and increasing more rapidly later, thanks to the availability of more precise star astrometry. After the publication of the Hipparcos and Tycho-2 star catalogues (Høg et al. 2000) the number of successfully recorded occultation events doubled in the decade 1991–2000 (see below for an evaluation of the current prediction accuracy). At the end of 2004 they summed up to 658, and today they continue to grow at a rate of about ~100 per year (Dunham & Herald 2005).

Despite the apparently large number of observed events, the extremely limited number of chords for most of the observed events results in diameters that remain poorly constrained. Also, the poor knowledge of shapes, rotational poles, and rotational phases at the epoch of the events often makes the interpretation of observations difficult. In practice, the use of occultation diameters found in current datasets[1] is not straightforward. As an example, Shevchenko & Tedesco (2006) were able to use just 57 occultation diameters for albedo determinations, among which only 18 have an estimated accuracy <5%. Shevchenko & Tedesco (2006) compared their "occultation albedos" against the albedos derived from IRAS observations in the thermal infrared, and against the presently available polarimetric albedos. However, because their data set contains occultation sizes only for asteroids larger than ~100 km, they were not able to explore any size-dependence of systematic errors of polarimetric and thermal infrared albedos.

Asteroid occultation observations – strongly encouraged by professional astronomers – are supported and secured by organizations grouping mainly volunteering amateurs, such as the International Occultation Timing Association (IOTA) or the European Asteroid Occultation Network (EAON), strictly cooperating with – and including – professional astronomers. Today, for stars currently included in the predictions of asteroid occultations (usually brighter than $V = 12$ and listed in UCAC or Hipparcos catalogues) the main source of prediction uncertainties resides in the accuracy of asteroid ephemerides (as discussed in the next section). Current uncertainties in the ephemerides of asteroids are rarely below ~0.5 arcsec (except for the largest bodies), corresponding to a displacement of ~350 km on the Bessel plane for an object at 1 AU from the Earth. The corresponding uncertainty on Earth has similar values, thus requiring a large number of appropriately spaced observers to increase the success probability of actually observing the occultation. Furthermore, in the coming years a slight degradation in the accuracy of occultation prediction can be expected, as a consequence of the uncertainty on the proper motions of stars.

The Euraster network data[2] show that in the year 2005, ~15% of the occultations observed under a clear sky was successful. This simple evaluation, however, does not include any information on the real position of the observers with respect to the predicted path on Earth, nor a distinction based on the presence of other degrading factors, such as a small flux drop, ephemeris uncertainty, short duration, and bad atmospheric conditions. It would thus be difficult and time-consuming to

[1] Interested readers should refere to the PDS asteroid occultation dataset (AOD), available from the NASA Planetary Data Systems Small Bodies Node: `http://www.psi.edu/pds/archive/occ.html`

[2] `http://www.euraster.net/results/index.html`

establish a more precise "average" success rate, since it depends on details that would need a case-by-case evaluation.

2.2. Current accuracy of asteroid ephemeris

The uncertainty in asteroid ephemeris can be evaluated by using information provided by the Bowell's Asteroid Orbit Database (ftp://ftp.lowell.edu/pub/elgb/astorb.html). The asteroid ephemeris uncertainty in this source is calculated by means of the procedure described by Muinonen & Bowell (1993), and it is expressed in terms of a small set of parameters, namely the CEU (Current Ephemeris Uncertainty), the PEU (Peak Ephemeris Uncertainty, and its date of occurrence), and the PEU over the following 10 years. All quantities are referred to the constantly updated epoch of the osculating orbital elements. The method adopted for the calculation of the CEU is considered to give meaningful estimates of the current ephemerides uncertainty for multi-opposition Main Belt Asteroids (MBAs) but, on the other hand, it is assumed to be unreliable for Earth crossers or asteroids with poorly determined orbits. Since we focus our study on the observation of occultations events of MBAs[3] we can assume that the CEU is representative of the uncertainty on the position at the epoch for which the osculating orbital elements are calculated.

Note that the CEU gives the length of the major axis of the uncertainty ellipse (usually very slender) representing the set of positions on the sky where the object can be found, at a 1-sigma confidence level. This axis is usually lying on the line of variations, or at a small angle with it. The main effect of this orientation will thus be a major uncertainty on the event epoch, not on the shadow path in the direction perpendicular to the asteroid shadow motion. This latter uncertainty is the dominating one as far as the observability of the occultation event is concerned; in fact, a shift in the predicted epoch can easily be compensated by an increased duration of the observation. It is thus likely that, by retaining the CEU as an estimate for the position uncertainty we are, in general, rather conservative. The exception is probably represented by a small number of large asteroids, having a smaller and more circular uncertainty ellipse, for which the CEU correspond more strictly to the prediction accuracy (Bowell, private communication).

Under these assumptions, we can then state that with a CEU larger than the apparent angular size of the asteroid, occultation events are difficult to predict. We therefore used the ratio of the CEU to the apparent angular radius θ of the asteroid (given by the ephemeris at the CEU epoch) as a statistical indicator for the current predictability of occultation events. For other epochs, of course individual CEU values will be different, but the overall statistics remain valid.

For our computations, the apparent sizes of asteroids θ were calculated from their IRAS diameters included in the Supplemental IRAS Minor Planet Survey (SIMPS) of Tedesco et al. (2002). Note that the SIMPS contains diameters for only the first ~2200 numbered asteroids. For the large majority of bodies in our sample, diameters are thus not known. For them, we estimated sizes from asteroid H magnitudes (which are included in the astorb.dat file) and an assumed geometric visible albedo via Eq. (1). However, the use of a constant albedo through the entire Main Belt would be an oversimplification: the albedo range for MBAs is very large (0.03–0.5), and it is known, for instance, that an inverse correlation between albedo and heliocentric distance exists. We used, therefore, the procedure described by Tedesco et al. (2005) to assign realistic albedos to the numbered asteroids not included in the SIMPS database.

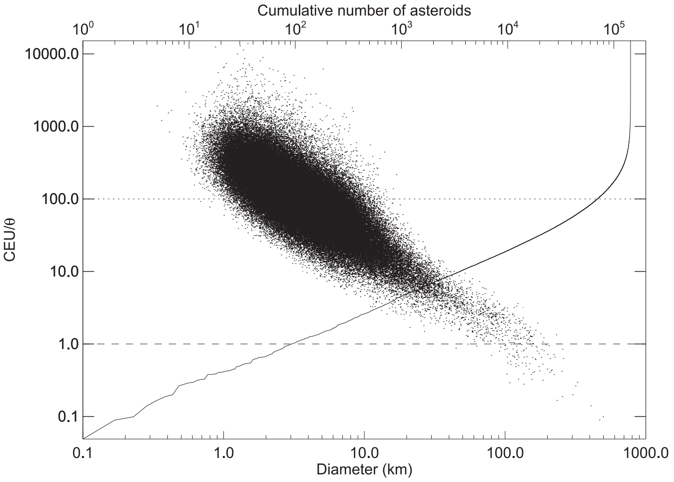

Fig. 1. Plot of the ratio CEU/θ versus the object diameter (in km, lower X-axis) for the numbered main belt asteroids known and observable at the date of writing: Feb. 20, 2007. The solid curve, superimposed on the plot, shows the cumulative number of asteroids (upper X-axis) which have a CEU/θ ratio smaller or equal than the corresponding value of the Y-axis on the left-hand side of the figure. The curve clearly shows that there are about 80–90 asteroids with a CEU/θ value smaller than 1. We expect that Gaia will improve the accuracy of asteroid ephemeris prediction by a factor ~100 (dotted horizontal line), increasing by almost three orders of magnitudes the number of asteroids for which CEU ~ θ.

According to this model (and using the numbering as in Tedesco et al. 2005), the main belt is divided into three zones: i.e. zone 2 for $2.064 < a < 2.50$; zone 3 for $2.50 < a \leq 2.82$; zone 4+5 for $2.82 < a < 3.278$, where a is the orbital semimajor axis of the asteroid. To assign an albedo, we first divide MBAs according to the zone to which they belong (2, 3 or 4+5) and to their mean apparent visible magnitude at opposition, given by $V(a, 0) = H + 5\log(a(a-1))$. Either the observed or the bias-corrected albedo distribution of Tedesco et al. 2005 (Table 3) is assigned to an asteroid, depending on whether the value of $V(a, 0)$, is <15.75 or ≥15.75, respectively. The same table gives the zonal distribution for four different classes of albedos: i.e. low ($0.020 < p_V \leq 0.089$), intermediate ($0.089 < p_V \leq 0.112$), moderate ($0.112 < p_V \leq 0.355$), and high ($0.355 < p_V \leq 0.526$). We then weight the number of asteroids according to the corresponding zonal albedo distribution and randomly distribute albedos uniformly within each of the albedo classes.

Figure 1 shows the resulting ratio CEU/θ for all numbered MBAs known and observable at the date of writing (astorb.dat file of Feb. 20, 2007) as a function of their diameter. We assumed that an asteroid is "observable" (in a broad sense) if it has a solar elongation ≥50°. The distribution shows that only very few asteroids (less than 90) – mainly bright MBAs with diameters larger than 50 km – have a ratio CEU/θ smaller than one. This roughly corresponds to the number of asteroids for which we can expect today to make reliable predictions of occultation events, or stated in another way, their occultations have a high probability of being observed inside the predicted shadow path. An improvement of occultation predictions is at present still possible through last-minute astrometry, aiming to constrain the

[3] According to Tedesco et al. (2005) we define MBA as those asteroids having orbits between the 4:1 and 2:1 mean-motion resonances with Jupiter, i.e., those with orbital semimajor axes between 2.064 and 3.278 AU and with modest inclinations (<25°) and eccentricities (<0.3).

trajectory of the asteroid with respect to the target star in the hours immediately preceding the event.

It is probable that only by the forthcoming all-sky astronomical surveys, which are capable of providing accurate multi-epoch astrometry of asteroids, will we be able to extend occultation studies of minor bodies to much smaller sizes, with the possibility of deriving occultation diameters for several tens of thousand of asteroids. The all-sky survey program that will have the best ever astrometric accuracy is the ESA mission Gaia, foreseen to be launched at the end of 2011.

However, the capability of systematic asteroid occultation observing campaigns of providing accurate size determination can be judged only if we can estimate the observational effort required. Thus, we will calculate in the following section the theoretical asteroid occultation rate for a given site on Earth, as a function of the asteroid diameter. Later, ephemeris and star position uncertainties will be taken into account to estimate the success rate of asteroid occultation measurements considering instruments with different performances, from those of a 20 cm telescope, up to the the case of a one-meter telescope equipped with a state-of-the-art CCD camera.

3. A numerical model for computing the occultation rate

In order to reproduce a realistic occultation rate for asteroids of different sizes, we need to estimate how many stars each asteroid occults in a unit time (e.g. one day). This is equivalent to the number of stars contained within the area on the sky swept by each asteroid in that time interval, and visible for a generic observer on Earth. This area can be easily calculated by taking the product between the apparent angular diameter of the asteroid, 2θ, and the distance on the sky between the two positions of the body calculated with the difference of one day. In order to calculate the density of stars falling within the area swept by each asteroid every day, we extracted the stellar density from the realistic galactic model used by Crosta & Mignard (2006). In this model the number of stars per square degree, as a function of their magnitude and their galactic coordinates, is expressed in terms of a set of Chebyshev polynomials, whose coefficients were determined by a fit to the measured star count. Note that, in practice, the number of stars occulted every day by each asteroid is much smaller than one, so to decide whether an occultation event take place or not, we generated a random number uniformly distributed between zero and one, and we took as positive events those cases where the random number was found to be greater or equal to the number of occulted stars. If the test was positive, the epoch of the occultation event was generated randomly inside the time step interval. We repeated this procedure every day over a period of 5 years, in order to consider different apparitions for each MBA, thus including different sky patches visited by each body.

We assumed our observer to be situated at the geocenter. This choice is not a loss of generality, since the overall statistics will be the same for any given site on the surface of the Earth. However, we divided by a factor of 2 the number of occultations obtained, to take into account the half celestial sphere visible in the presence of a theoretical, geometric flat horizon. One should note that this correction could be different (and probably larger) in practice, due to the difficulty of observing events at low sky elevations. Moreover, to calculate asteroid positions we solved the unperturbed two-body problem: planetary perturbations represent a level of refinement not required for our purposes. As

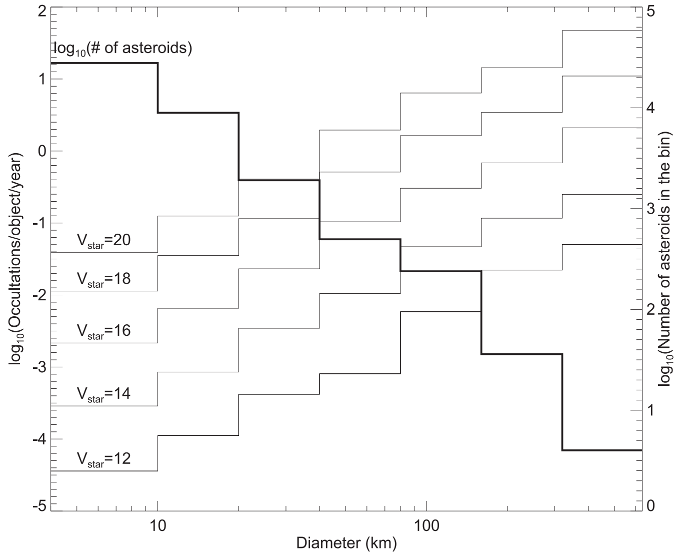

Fig. 2. Number of occultations per object during one year, as a function of the asteroid diameter (thin lines). These statistics are valid for a single site and are computed for different values of the limiting magnitude as indicated by the legend. The thick curve (scale on the right-hand side y axis) represents the number of asteroids for each bin in the simulated population. The product of this last number with each rate/object curve provides the total number of occultations per year for a given size range.

for asteroid diameters, the same values assigned as described in Sect. 2.2 were adopted. Finally we computed, for each of the simulated events, the maximum duration of the occultation τ_o using the object angular diameter and apparent motion at the occultation epoch, the asteroid's apparent magnitude m_a, and the magnitude drop during the occultation.

The obtained results, of course, have a statistical value. They are presented in Fig. 2, where we show the average number of occultation events that occur for a single site and a single object, during one year. It is clear that including faint stars strongly enhances the occultation rate. In fact, we find that any object larger than 20–30 km occults a 20th magnitude star at least once per year.

However, the theoretical occultation rate that we have computed so far represents a theoretical value based on geometry only. It does not take into account the "observability" of each event, that is the capability of a given instrument to reveal the temporary flux drop produced by the occultation. Furthermore, since the scientific goal that interests us is the impact of occultation observations on asteroid size determination, we restricted our study to events from which the the size of an asteroid can be determined within a given uncertainty level.

We investigate the observability of the events by comparing portable equipment, similar to those currently used by most observers in present days, to a completely different situation, i.e. that of a fixed 1-m class telescope equipped with a cooled CCD camera. With the larger instrument, much fainter stars (and much smaller magnitude drops) can be measured, extending the investigation of occultation events toward smaller asteroids. As we will see, this will in turn allow the exploitation of improved asteroid ephemeris and stellar astrometry.

4. The measurement of an occultation: limits imposed by the instrument

The observation of an occultation event consists of the measurement of the temporal flux variation, by means of rapid photometry, of a single source composed by the unresolved images of the star and the asteroid. A flux drop occurs during the occultation when only the asteroid hiding the star is observable. The duration of the occultation is typically of a few seconds for large (50–100 km) asteroids, and the most favorable events, of course, involve a bright star and a faint asteroid. In this case, the flux drop can reach several magnitudes. We will use the symbols $m_{\rm a}$, $m_{\rm s}$, and $m_{\rm sa}$ to indicate respectively the magnitude of the asteroid, the magnitude of the star and the magnitude of the unresolved source star + asteroid immediately before and after the occultation. For the corresponding measured fluxes we adopt the notation $F(m_{\rm a})$, $F(m_{\rm s})$, and $F(m_{\rm sa})$, respectively.

To obtain useful size estimates of asteroids from the observation of an occultation event, the use of performing equipment, capable of exposure times considerably shorter than the maximum event duration, and sensitive enough to allow accurate photometric measurements of the flux drops is required. Still, a single observation of an occultation will not yield, in general, a direct measurement of the object's size. This is because the measured occultation chord does not necessarily correspond to the largest projected size of the asteroid occulting the star. Moreover, in general, the three-dimensional shape of the occulting asteroid and its orientation should be known for a meaningful interpretation of the results obtained from occultation measurements.

As for the definition of the signal to noise ration (*S/N*) of the photometric measurements of the occultation event, we adopt the ratio between the amplitude of the flux drop ($F(m_{\rm as}) - F(m_{\rm a})$) and the photometric uncertainty associated with the flux during ($\sigma_{F({\rm a})}$) and outside ($\sigma_{F({\rm as})}$) the occultation:

$$S/N = \frac{F(m_{\rm as}) - F(m_{\rm a})}{\sqrt{\sigma^2_{F({\rm a})} + \sigma^2_{F({\rm as})}}}. \quad (2)$$

For a given instrument setup and for each event, the available flux and the imaging equipment characteristics determine the corresponding photometric uncertainty.

After having chosen an instrument set (see below) we thus computed – for each event – the minimum exposure time $t_{\rm m}$ providing *S/N* ∼ 3. Assuming that the absolute timing is available for the CCD frames with negligible uncertainty, we take $t_{\rm m}$ as the estimate of the occultation timing uncertainty, i.e. $\sigma_\tau = t_{\rm m}$. Since the maximum transit duration is proportional to the object diameter D, the ratio $\sigma_{\rm D} = \sigma_\tau/\tau_{\rm o}$ (where $\tau_{\rm o}$ represent the maximum occultation duration, on the center line) will represent the relative accuracy expected for the diameter determination.

One should note that, according to the *S/N* definition above, an exposure time yielding $S/N \sim 3$ will always be found, but in some cases it will be comparable to – or significantly longer than – the occultation duration. However, we will consider as "non–interesting" those events having a relative uncertainty on the final diameter determination $\sigma_{\rm D} > 40\%$. By adopting this constraint, "unobservable" events (i.e. those requiring $t_{\rm m} \leq \tau_{\rm o}$) are discarded. We then classified the remaining events as function of $\sigma_{\rm D}$.

Of course, discarded events could still be of scientific value, for example for the search of asteroid satellites, however in this study we do not address in detail this otherwise interesting subject.

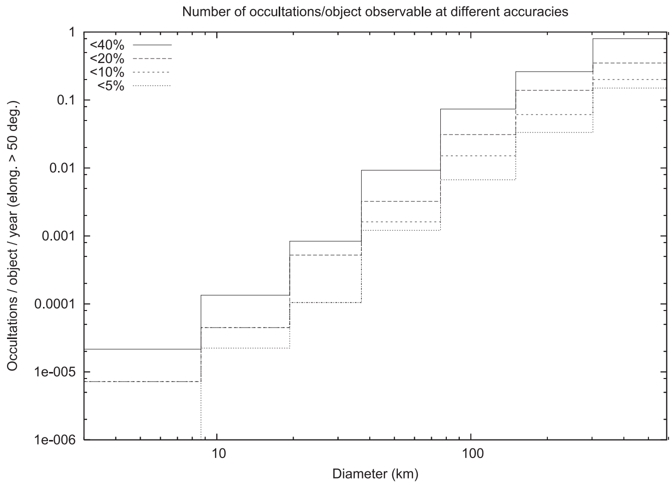

Fig. 3. Number of observable occultations per object during one year, as a function of the asteroid diameter, for a 20 cm telescope equipped with a cheap fast-readout camera. These statistics are valid for a single site and are computed for different values of the maximum relative uncertainty on size, $\sigma_{\rm D}$, as indicated by the legend.

The predictions of occultation events that are currently distributed to observers include target stars not fainter than $V = 12$–13, thus reducing the occultation rate at about 0.1 events/objet/year even for the largest bodies. As a further selection, requirements on the minimum occultation duration and magnitude drop mean that to only the ∼1000 asteroids larger than ∼40 km are taken into account. We ran a first simulation considering the performance of a 20 cm, f/10 telescope equipped with a rapid, uncooled video camera (30 frames per second, exposure 1/30 s) with rather high readout noise (15 e$^-$/pixel). We considered a CCD operating at 290° K, at a pixel scale ∼0.9 arcsec/pixel, with a seeing FWHM around 1 arcsec. Sky background was assumed to be $V = 12$ mag/arcsec^2. With the high CCD frame rate imposed, the dominating noise source is not dark current, but the read-out noise. The sensor temperature is thus of secondary importance. By using aperture photometry with an optimized aperture size, we obtain the results shown in Fig. 3. The integral of the occultation rate for objects $d > 40$ km yields no more than a few events observable at high accuracy each year for a single site. This appears to be fully consistent with the present observation statistics.

The simulation has also been run considering a 1-m telescope coupled to a low-noise, photon-counting CCD camera. These cameras, based on an electron multiplier in the read-out register, operate at very low temperatures (∼180° K) and presently represent the state-of-the-art technology for astronomical use. Considering a larger pixel (22 μm) and a faster focal ratio (f/6), the image scale (0.76 arcsec/pixel) is not far from that of the 20 cm telescope equipped as above. Read-out noise is considered to be (1 e$^-$/pixel); all other conditions are not varied relatively to the previous numerical experiment. The increase in camera performance and photon flux provides an improvement in the number of events at a given size range, by a factor of 1 to 2 orders of magnitude (Fig. 4).

In the following section, we use the statistics obtained with these two extreme instrument setups as a reference, and we include in our analysis the contribution of uncertainties on both asteroids and star positions.

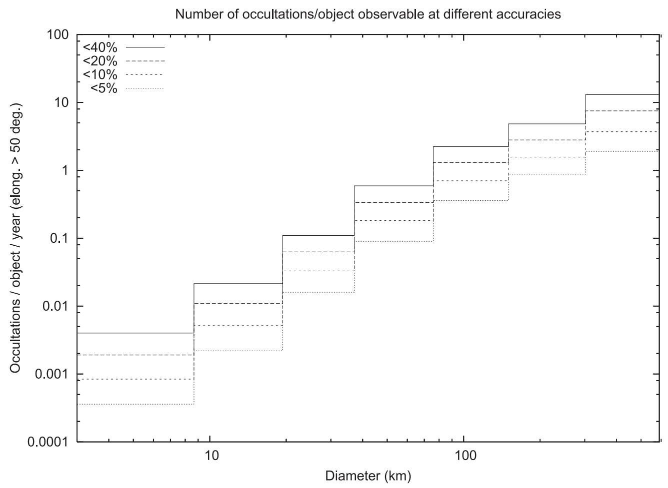

Fig. 4. Number of observable occultations per object during one year, as a function of the asteroid diameter, for a 1 m telescope equipped with a cooled, low-noise, fast-readout camera. The histogram is computed, as in Fig. 3, for different values of σ_D.

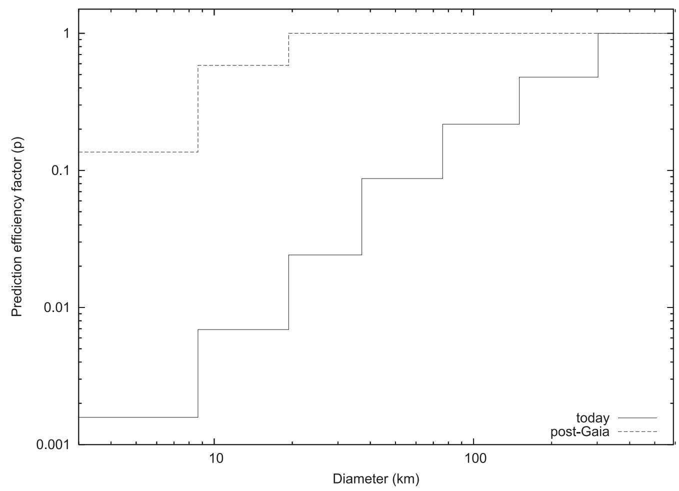

Fig. 5. Prediction efficiency factor for "present day" and "post Gaia" accuracies on asteroid ephemeris and star positions.

5. Role of astrometric accuracy

Up to now we have discussed the statistics of occultation events and their observability as if we were able to know asteroid and star positions with infinite accuracy. In reality, they are affected by astrometric uncertainties. As a consequence, in principle, an observer has to monitor a larger number of candidate events in order to reach the count of the positive occultations predicted by the theoretical rate. The number of events that need to be monitored can be estimated by assuming as "impact section" for each asteroid the apparent size given by its diameter plus the width of the ephemeris uncertainty at the occultation epoch. Adding an uncertainty on the star position will further increase the "impact section".

To take this effect into account, for each event we computed an estimate of the total uncertainty on the occultation path as: $\sigma_{\rm path} = \sqrt{\sigma^2_{\rm CEU} + \sigma^2_*}$, where $\sigma_{\rm CEU}$ is assumed to be equal to the CEU for the asteroid discussed above, while σ_* represents the uncertainty on the star position. The prediction lists usually diffused to the amateur community include the stars of the catalogues FK6 (I+III), Hipparcos, Thyco 2, and UCAC2 for stars of magnitude $10 < V < 12.5$ (Dunham & Herald 2005). For stars with $V < 10$ the UCAC2 positions, if available, are used to replace those given in the Thyco 2. A reasonable estimate of star position uncertainties is of about 1 mas for the FK6, 10 mas for the Hipparcos, 70 mas for Tycho 2, and 20 mas for the UCAC2, for sources in the range of magnitudes $V \sim 10$–14, whereas the uncertainty is of about 40 mas outside this range (Zacharias et al. 2004). For each simulated event we thus attribute to the occulted star an uncertainty typical of Thyco-2 or UCAC2 stars, with equal probability (this roughly reflects the current distribution of catalogue exploitation for predictions, see Dunham & Herald 2005).

In a second step, we compute the ratio $p = 2\theta/\sigma_{\rm path}$, (we recall here that 2θ is the apparent asteroid diameter) averaged this over all the events occurring in a given diameter range. This can be considered to be a "prediction efficiency factor". If we assume that the uncertainties on asteroid ephemeris and star positions are Gaussian distributed, then $\sigma_{\rm path}$ represent the standard deviation of the total uncertainty on the occultation path, and $p = 1$ implies a 1-sigma confidence level (68%) of actually being able to observe the occultation event. This could certainly be considered as an event deserving an observation, with high success probability. In reality, the CEU can follow a more complex distribution, and the $\sigma_{\rm path}$ has to be taken as a rough estimate of dispersion rather than a true Gaussian deviation. Thus p cannot be considered more than a reasonable guess of the success probability for each event. On the other hand, one could also note that $1/p$, for $p < 1$, is an estimate of the number of events that should be observed to witness one positive event, on average. For those events with $p \sim 1$ the occultation prediction is accurate and, in those cases where we have $p \gg 1$ the occultation path is known so accurately that it could be even possible to determine the position of the chord relative to the object barycenter.

In the case of the after-Gaia statistics, different assumptions are needed for both CEU and σ_*. Since the dependence of the ephemeris accuracy on the asteroid size should preserve the general features shown in Fig. 1, we simply assume an improvement factor of 100 in the ephemeris uncertainty of each object, i.e. a post-Gaia value of the CEU reduced by a factor of 100 with respect to the present one. This is a reasonable assumption in agreement with different analytical estimates and simulations of the improvements in asteroid orbital elements that Gaia will allow (Virtanen et al. 2004; Muinonen et al. 2004; Hestroffer & Berthier 2005). We also assume that the end-of-mission accuracy on star positions (in mas) can be represented by:

$$\sigma_*(\text{mas}) = 4 \times 10^{-2}\ \mathrm{e}^{0.44(V-20)+1.35}, \quad V > 12 \tag{3}$$

where V is the star magnitude. A value $\sigma_* = 17\ \mu$as is obtained at $V = 15$[4]. In this case σ_* saturates to the value reached at $V = 12$ for brighter stars.

The result of the computation of p is given in Fig. 5 for the present and post-Gaia asteroid ephemerides and star position uncertainties. It is clear that the present occultation predictability appears reasonable ($p > 0.1$) for the largest Main Belt asteroids only ($D > 100$ km), but in the "post-Gaia" era an equivalent accuracy can be reached down to ~15 km.

[4] This accuracy is obtained from simulations of the on-board detection (Arenou & Lim 2003) and from the expected mission performances.

6. Discussion and perspectives

Let us consider a 10–20 km asteroid as a representative example. By multiplying the number of objects in the model at the corresponding bin (10^4) with the current number of occultation per object per year for a small instrument (Fig. 3), one finds that about 0.1 occultations/year occur at a 5–10% level of size accuracy for a single site. Furthermore, prediction efficiency (Fig. 5) is so low (~0.002) that several hundred events should be observed (on average) to obtain one success. It is thus simply impossible, with this equipment, to obtain accurate asteroid occultation diameters in this size range.

On the other hand, for asteroids with $D \simeq 100$ km the predictability is rather good (~0.2) and bright star occultations ($V < 12$), observable with the same equipment, occur ~0.01 times per object per year (at high accuracy). Monitoring several objects by several sites will be sufficient to obtain, over a period of years, a sample of observed positive events. This is compatible with a lower limit at ~100 km for the currently available occultation-derived diameters.

One should note that observing occultations with larger diameters can improve the accuracy of photometry, and increase the number of candidate events by including smaller flux drop. However, with present days asteroid ephemerides uncertainty, the predictability of the occultation events remains poor, thus preventing efficient observations at small sizes.

Much more interesting is the case of the post-Gaia scenario, in which asteroid orbits are improved by a factor ~100 and stellar astrometry makes a revolutionary step forward for star brightness down to $V \sim 20$. In fact, in this case the prediction efficiency remains $p = 1$ (or larger) down to a diameter of 20 km. This implies not only a perfect predictability of the events themselves, but also an uncertainty lower than the width of the occultation shadow on Earth. In other words, observers of occultations of asteroid with $D > 20$ km will have some indications of the position of the chord relative to the projected shape of the object. This represents a major change in the way asteroid occultations can be approached, and could make them an even more powerful instrument for asteroid studies.

In Fig. 6 we provide the distribution of the occultation durations and star magnitudes for events involving asteroids in the 15–20 km range. Comfortably long occultations and relatively bright stars are still present even toward the small end of the asteroid size distribution for predictable occultations. Also, we computed that ~70% of the events have a flux drop larger than 1 magnitude. However, in order to exploit the full potential of asteroid occultations in the post-GAIA era, more powerful instruments must be devoted to this activity. Figure 4, shows that – by multiplying the predicted rate by the corresponding number of objects – 20 to 50 occultations/year are theoretically observable with the highest success probabilities for each 10–20 km object, by using a 1 m telescope, with diameter uncertainties <10%. All known objects in this size range will lie within the CEU/$\theta \sim 1$ limit if a two-orders of magnitude orbit improvement is reached after Gaia (Fig. 1). This result has impressive implications, since it means that, in principle, all objects larger than that limit could be measured by monitoring occultations. Furthermore, the relatively large number of events could allow a single site to collect good statistics for each object, thus allowing an accurate diameter determination. In practice, the large number of asteroids involved (10^4) implies that a few hundred events should be monitored each day. Of course, this is a huge effort for a single instrument or team, but it remains feasible if spread over a few years or if several sites with similar instruments are cooperating.

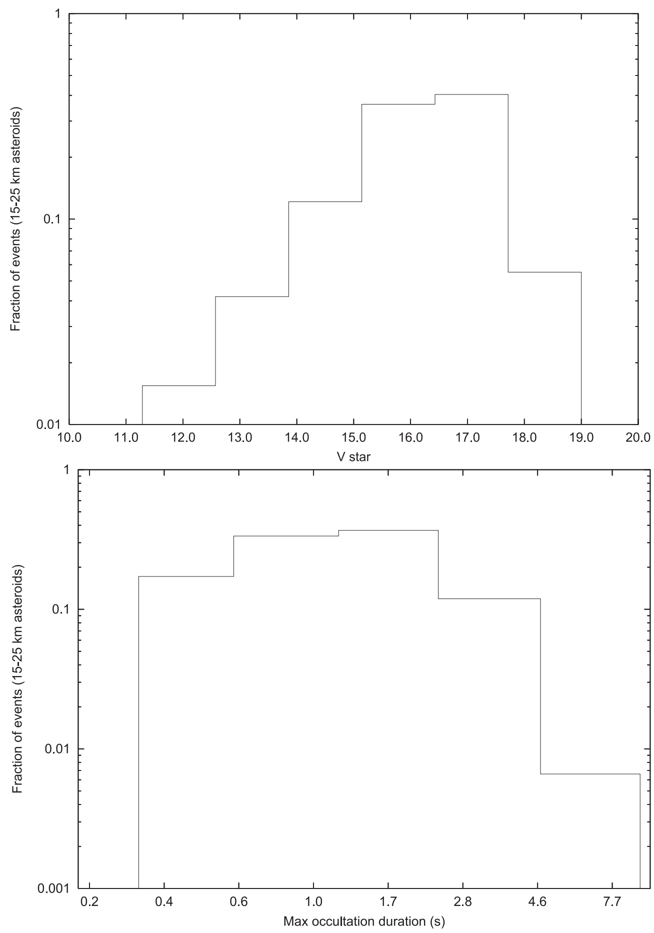

Fig. 6. Distributions of the occulted star V magnitude (upper panel) and of the duration of the events (lower panel) for all occultations involving asteroids in the diameter range 15–25 km.

The number of events at smaller asteroid diameters will still be interesting, although completeness will be out of reach. A selection of objects with the lowest ephemerides uncertainty will represent a good sample for extending size distributions toward km-sized asteroids.

Gaia will not only provide accurate astrometry, but also photometry and spectroscopy. Experiments of lightcurve inversion from spare data (Cellino et al. 2006) show that shapes and rotation vectors could be retrieved for potentially all the main belt asteroids from Gaia observations. Earth-based surveys such as Pan-STARRS could provide a significant contribution, although photometric accuracy will be much lower. However, the availability of shapes and physical ephemerides will allow extraction of a maximum of information from occultation studies, since the orientation of the asteroid as projected on the sky at the occultation epoch will be known.

Several asteroid satellites will also be discovered and their relative obits determined, thus enriching even more the potential of further occultation events.

In conclusion, all the statistical arguments presented in this study indicate that, when Gaia observations will be available, a small set of dedicated 1 m telescopes will be able to provide – within a few years – a complete census of asteroid diameters down to ~10 km in size. The equipment foreseen to be used for future asteroid occultation studies such as fast, low noise CCD cameras, represent the state-of-the-art present-day technology, but it is reasonable to predict that in ∼10 years from now (the after-Gaia era) even more optimized and performing devices will be available. Today, low noise CCD cameras are beginning to be used in professional occultation equipment (Souza et al. 2006) and we can expect that they will encounter a wide diffusion in the future. Considering the constant progress made in parallel by instruments accessible to the non-professional community, it is also highly likely that the role of dedicated amateur astronomers will continue to be a precious source of scientific information for asteroid studies.

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