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Asteroid models from combined sparse and dense photometric data

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ABSTRACT

Aims. Shape and spin state are basic physical characteristics of an asteroid. They can be derived from disc-integrated photometry by the lightcurve inversion method. Increasing the number of asteroids with known basic physical properties is necessary to better understand the nature of individual objects as well as for studies of the whole asteroid population.
Methods. We use the lightcurve inversion method to obtain rotation parameters and coarse shape models of selected asteroids. We combine sparse photometric data from the US Naval Observatory with ordinary lightcurves from the Uppsala Asteroid Photometric Catalogue and the Palmer Divide Observatory archive, and show that such combined data sets are in many cases sufficient to derive a model even if neither sparse photometry nor lightcurves can be used alone. Our approach is tested on multiple-apparition lightcurve inversion models and we show that the method produces consistent results.
Results. We present new shape models and spin parameters for 24 asteroids. The shape models are only coarse but describe the global shape characteristics well. The typical error in the pole direction is ∼10–20°. For a further 18 asteroids, inversion led to a unique determination of the rotation period but the pole direction was not well constrained. In these cases we give only an estimate of the ecliptic latitude of the pole.

Key words. minor planets, asteroids – techniques: photometric – methods: data analysis

1. Introduction

The lightcurve inversion method is a powerful tool to derive basic physical properties of asteroids (rotation periods, spin axis orientations, and shapes) from their disc-integrated lightcurves (Kaasalainen & Torppa 2001; Kaasalainen et al. 2001, 2002a). As has been proven by space probes, laboratory models, and adaptive optics images (Kaasalainen et al. 2001, 2005; Marchis et al. 2006), models derived by this method are good approximations of the real shapes of asteroids. In the standard approach, the lightcurve inversion method is applied to a set of typically tens of lightcurves observed during at least three or four apparitions – only then can the spin state and the corresponding shape be derived uniquely. Kaasalainen (2004) showed that to derive a unique physical model it is not necessary to have dense lightcurves – i.e., brightness measurements that densely sample brightness variations during one revolution. Lightcurve inversion can also be used on so-called "sparse data" that only sparsely sample brightness variations. Typically, sparse data consist of at most a few measurements per night. A set of more than about one hundred calibrated measurements

sparse in time is fully sufficient for modelling as long as the photometric accuracy of the data is better than ∼5% (Ďurech et al. 2005, 2007). Sparse data that will be available from all-sky surveys such as Pan-STARRS, LSST or Gaia are extremely time-efficient in obtaining new asteroid models compared to the standard approach of observing individual objects. However, accurate sparse data are not yet available, and all asteroid models based on photometry were derived by inversion of standard dense lightcurves (e.g., Kaasalainen et al. 2002b, 2004; Torppa et al. 2003).

The main challenge to be solved when inverting sparse data is the correct determination of the rotation period. Because there is no direct information about the rotation period in the data, the correct value has to be found by scanning the whole interval of possible values and looking for the best fit. This is a time consuming process that yields a unique period solution only if there are enough data points (more than about a hundred) and if the data photometric accuracy is better than about ∼5%. If these conditions are not fulfilled, there is usually more than one period with the same fit to the data (for more details see Ďurech et al. 2005, 2007). On the other hand, just one

ordinary dense lightcurve usually defines the possible range of the rotation period well. As has been shown by Kaasalainen & Ďurech (2007), noisy sparse data can be combined with a few ordinary lightcurves and such combined data sets give in many cases a unique solution. When combining sparse data with dense lightcurves, inversion leads to a unique solution for the period even if sparse data are noisy and the number of dense lightcurves is small.

In this paper we present new asteroid models derived from combined data sets. We used sparse data from the US Naval Observatory, Flagstaff (USNO) and dense lightcurves from the Uppsala Asteroid Photometric Catalogue (UAPC, Lagerkvist et al. 2001) and from the archive of the Palmer Divide Observatory (PDO).

We also present the results of various tests that estimate the errors in pole directions and show that our results are reliable.

2. Combined data

As stated above, combining sparse and dense photometry can yield a unique model even in cases when neither sparse data nor lightcurves would be sufficient for modelling alone. Sparse data typically cover more apparitions carrying thus information about brightness variations for different geometries. Dense lightcurves, on the other hand, well define the rotation period and enable us to narrow down the search interval.

2.1. Standard lightcurves

Most of the photometric lightcurves we used were archived in UAPC. Apart from this source, we also used the database of lightcurves of more than 200 asteroids observed at the Palmer Divide Observatory[1]. We also included new observations of (34) Circe and (416) Vaticana. Circe was observed by F. Pilcher (Las Cruces, New Mexico, 35-cm telescope) in 2007 and by M. Fauerbach and S. A. Marks (Egan Observatory, Florida, 40-cm telescope) in 2007/2008. Vaticana was observed in 2006/2007 by M. Fauerbach (Egan Observatory, Florida, 40-cm telescope) and by S. Fauvaud, J.-M. Vugnon, and M. Fauvaud (Pic du Midi Observatory, France, 60-cm telescope).

2.2. Sparse data

Sparse photometric data accurate enough to provide a unique solution of the inverse problem are not yet available. However, to obtain new asteroid models we need not wait for them – we can use already available less accurate sparse photometry and combine it with dense lightcurves. A huge amount of sparse photometry is provided each night by many observatories that carry out astrometric observations. Unfortunately, as these measurements are not intended for further photometric analysis, their quality is usually very poor from the photometric point of view. The only exceptions (as far as we know) are measurements made at the USNO. We estimated the typical error of these data to be ~0.08–0.1 mag. The database of astrometric measurements from the USNO is available at the Asteroid Dynamic Site[2]. The data are available for the first 2000 numbered asteroids; they cover

about five years and typically consist of 50–200 individual measurements.

3. Tests

Because our aim was to derive asteroid models based on noisy sparse data and a small number of dense lightcurves, we carried out various tests to determine how accurate and reliable the obtained results were.

For our tests, we used the Database of Asteroid Models from Inversion Techniques (DAMIT)[3] now containing about 80 models derived from dense lightcurves. From DAMIT, we selected 60 asteroids that had enough sparse USNO data (more than 30 points). From the full set of dense photometry for each asteroid, we randomly selected individual lightcurves and combined them with the sparse data to create test data sets. For each asteroid, we created twelve different test data sets: one set with six lightcurves ($N_{lc} = 6$) from three apparitions, two sets with four lightcurves ($N_{lc} = 4$) from two apparitions, three sets with two lightcurves ($N_{lc} = 2$) from one apparition, and six individual lightcurves ($N_{lc} = 1$). Each data set also contained sparse USNO data. We then processed the test data sets the same way as the real data using the lightcurve inversion method (Kaasalainen 2004; Ďurech et al. 2005, 2007). The inversion algorithm derives a spin/shape model that minimizes the difference between observed and computed brightness. The minimization process can use dense and sparse data and we decided to weight the sparse data one third of the dense data. We used information about rotation periods and reliability codes listed in the Minor Planet Lightcurve Parameters database[4]. All asteroids from DAMIT have reliability codes 3 or 4. For each asteroid, we scanned an interval of periods centered at the reported period P with a width of ±1% and ±3% of P for reliability codes 4 and 3, respectively[5].

As expected, in most cases the limited amount of data was not sufficient to provide us with a unique solution – out of the 60 asteroids and 12 tests for each asteroids there were only 22 asteroids that had at least one unique solution. A *unique solution* was defined as follows: the best period had at least 10% lower χ^2 than all other periods from the scanned interval and, for this period, there was only one pole solution with at least 10% lower χ^2 than the others (with a possible mirror solution for $\lambda \pm 180°$).

We compared the obtained results with the models from DAMIT based on full lightcurve sets – the DAMIT models were taken as established. Results listed in Table 1 report the maximum difference Δ_{max} (in degrees) between the pole solution for the corresponding test data set with N_{lc} lightcurves and the original DAMIT solution based on a full set. We do not report the difference in periods, since it was always within the expected error (the two exceptions (125) Liberatrix and (192) Nausikaa are discussed below). If there were more solutions for different test data sets with the same N_{lc}, we chose always the largest difference between the true model and the derived one. Thus Table 1

Table 1. Test asteroids. For each asteroid, the table gives the Δ_{max} difference between the pole direction of the model derived from the full set of lightcurves and that derived from N_{lc} lightcurves and N_{sp} sparse points. In most cases, there are two values of Δ_{max} for one N_{lc} column. This corresponds to two models with $\lambda \pm 180°$ ambiguity in the pole direction.

Asteroid	N_{sp}	Δ_{max} (deg)							
		$N_{lc} = 6$		$N_{lc} = 4$		$N_{lc} = 2$		$N_{lc} = 1$	
3 Juno	171			15					
15 Eunomia	147	10							
32 Pomona	128			25				13	
39 Laetitia	181					15			
43 Ariadne	114	2							
44 Nysa	170	11							
63 Ausonia	137	4	3	5	7	9	8	13	12
107 Camilla	116	9	2	2	5	2	9		
125 Liberatrix	105							146	145
130 Elektra	113	4		12				7	
158 Koronis	135	26	23						
192 Nausikaa	174	75							
201 Penelope	115	6	3						
277 Elvira	93	7				8	5		
283 Emma	96	4	6	6	6	50	8	3	6
306 Unitas	130	2	3	5	4	6	5	7	8
382 Dodona	121	8		21	8	20	23	21	21
532 Herculina	135	15							
584 Semiramis	97	18		20		15			
665 Sabine	118	11		13				16	
675 Ludmilla	159	13	10	16	19	20	19	21	23
1627 Ivar	68			18					

Table 2. Difference in the pole ecliptic latitude β for multiple-pole solutions with a unique determination of the period but unconstrained value of the pole longitude λ and the variation in β less than 50°. For each asteroid, the table gives the $\Delta\beta_{max}$ difference between the pole latitude of the model derived from the full set of lightcurves and the mean value of β for non-unique models derived from N_{lc} lightcurves and N_{sp} sparse points.

Asteroid	N_{sp}	$\Delta\beta_{max}$ [deg]			
		$N_{lc} = 6$	$N_{lc} = 4$	$N_{lc} = 2$	$N_{lc} = 1$
3 Juno	171	29			
15 Eunomia	147		7	7	4
19 Fortuna	213	4			
20 Massalia	143		0		
32 Pomona	128	21	18	18	20
39 Laetitia	181		28		
44 Nysa	170		11		0
107 Camilla	116		4	4	3
125 Liberatrix	105		21	21	
129 Antigone	149	3	1	1	1
135 Hertha	102	21		0	
158 Koronis	135		4	4	5
192 Nausikaa	174		64		
277 Elvira	93		14		
306 Unitas	130			0	19
382 Dodona	121				0
584 Semiramis	97		22	22	
665 Sabine	118			0	24
1223 Neckar	96				0
1627 Ivar	68				0

reports the worst-case scenario results. From the values of Δ_{max}, we can estimate the reliability of pole/period solutions based on combined data. If no unique solution was found for any test data set with a given N_{lc}, we had nothing to compare and the value of Δ_{max} for the corresponding N_{lc} is missing. If there are two values of Δ_{max} in one N_{lc} column, they correspond to two established models ($\lambda \pm 180°$ ambiguity) in DAMIT.

Apart from the asteroids Liberatrix, Nausikaa, and Emma that are discussed below, we obtained a correct solution for the period and a pole for all test asteroids. The pole difference was never greater than ~25°. This justifies the use of combined data sets for lightcurve inversion and gives credibility to new asteroid models derived in Sect. 4.

(125) Liberatrix. This case revealed the limits of combined data sets. Although the sparse data consisted of more that one hundred points, in one test case these data combined with only one dense lightcurve led to an incorrect period, thus to an incorrect pole direction and an incorrect shape model. The correct rotation period, 3.9682 h, gave the second best χ^2 that was ~10% higher than the lowest χ^2 for the period of 3.9491 hr. There were no unique period solutions for the other eleven test data sets.

(192) Nausikaa. This asteroid gave a large difference between the solution derived from the combined data set (six lightcurves from three apparitions and sparse data) and the original solution by Kaasalainen et al. (2002b). However, the new analysis of all available lightcurves showed that the published result is only one of further possible solutions. More observations are needed to derive a unique model.

(283) Emma. One solution based on sparse data and two lightcurves from one apparition gave a pole solution as far as 50° from the correct one. This again shows that data from one apparition with only one or two lightcurves may lead to an incorrect solution. However, all other combinations of test data led to solutions that were very close (difference in pole directions <10°) to the true one.

(675) Ludmilla. The model of asteroid Ludmilla published by Torppa et al. (2003) was inconsistent with new observations carried out by Bernasconi[6]. We derived a new model (available now in DAMIT) that was fully consistent with all available lightcurves and also with sparse data. Values of Δ_{max} in Tables 1 and 3 are based on the new model with the spin axis direction (in ecliptic coordinates) $\lambda = 49°$, $\beta = 74°$ (mirror solution $\lambda = 196°$, $\beta = 49°$) and the sidereal rotation period $P = 7.715486$ h.

We emphasize that DAMIT contains updated versions of asteroid models; several models published in the papers cited above have changed, some of them significantly. This is mostly due to updated data sets, but we note that software errors and/or too narrow ranges for the initial period have affected the quality of some models, whose corrected versions now appear in DAMIT. Thus, while citing the original paper in which the model for a given asteroid appears, one should always download the updated model information from DAMIT and not use the obsolete information given in the paper. Shortly, we will submit an online paper describing the database.

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[6] http://obswww.unige.ch/~behrend/page2cou.html#000675

Table 3. The same test as in Table 1 but the number of sparse data points was reduced to $N_{sp} = 50$.

Asteroid	Δ_{max} (deg)					
	$N_{lc} = 6$		$N_{lc} = 4$		$N_{lc} = 2$	
2 Pallas					95	98
15 Eunomia			19		40	
21 Lutetia					41	37
39 Laetitia	16		12		28	
43 Ariadne	2		1			
44 Nysa	9				104	
63 Ausonia			3	2	7	9
107 Camilla	1	4	9	7	3	13
130 Elektra			7		2	
201 Penelope	5	8				
277 Elvira	11	10	22	11	8	2
283 Emma	3	3	9	6		
306 Unitas			4	2		
311 Claudia	22	28			32	37
382 Dodona	12	2	27	14	20	19
532 Herculina	14				7	
534 Nassovia			20	18		
584 Semiramis	9					
624 Hektor	7					
665 Sabine					25	
675 Ludmilla	10	15	8	17	87	96
1223 Neckar	11	5				
1627 Ivar			18		19	

Table 4. The same test as in Table 2 but the number of sparse data points was reduced to $N_{sp} = 50$.

Asteroid	$\Delta\beta_{max}$ (deg)		
	$N_{lc} = 6$	$N_{lc} = 4$	$N_{lc} = 2$
15 Eunomia	6		3
23 Thalia			48
32 Pomona			33
44 Nysa			97
129 Antigone		7	
158 Koronis	6		
277 Elvira			14
311 Claudia		4	
382 Dodona			22
511 Davida			5
532 Herculina		24	17
584 Semiramis		22	
665 Sabine	24	23	14

different data sets with the same N_{lc}. One test yielded a unique solution and was listed in Table 1, while the other fullfiled the conditions mentioned above and was listed in Table 2. The only asteroid showing a large discrepancy between β values of the published and derived model is again Nausikaa. All other asteroids have differences in β lower than $\sim$30°, which is enough for a rough determination of the obliquity.

3.1. Determination of the pole ecliptic latitude β

Even if the rotation period of an asteroid can be determined uniquely, there can be more than one independent pole solution for this period that fit all observed data well. We have noticed that often in such cases, there are more possible pole solutions with very different values of the ecliptic longitude λ but very similar values of the ecliptic latitude β – thus we can take β as a well determined parameter[7]. Although we cannot determine the ecliptic longitude λ and the corresponding shape model, the ecliptic latitude β of the spin axis is an important physical parameter. For asteroids with small or moderate inclinations, β directly gives the obliquity – the angle between the rotation axis and the normal to the orbital plane.

Using the same test data sets and inversion procedure as in the previous test, we selected such solutions that had no unique shape and pole, but a unique period and poles with β values within an interval of 50°. Table 2 lists results for 20 such multiple pole solutions and reports the maximum difference $\Delta\beta_{max}$ between the mean value of β for models based on our test data set and β of the original model. This difference was so small in some cases that $\Delta\beta_{max}$ was zero after truncation. Again, this is the worst-case scenario, which means that we report the largest values if there are more solutions for different test data sets with the same N_{lc}. If there were more solutions for the period, or the derived poles for a unique period were different by more than 50° in β, the value of $\Delta\beta_{max}$ is missing. In some cases (for example (32) Pomona or (107) Camilla), there are values of Δ_{max} in Table 1 and $\Delta\beta_{max}$ in Table 2 for the same N_{lc} column. This means that the values of Δ_{max} and $\Delta\beta_{max}$ were derived for

3.2. Limited amount of sparse data

The number of sparse data points for asteroids from our tests was usually more than one hundred (see Table 1). On the other hand, some sparse data used to derive new models in Sect. 4 consisted of fewer than one hundred points (see Table 5). In order to test what happens if we use less sparse data and to find out how reliable the derived models are, we reduced the amount of sparse data. We used the same test data sets as in Sect. 3 but instead of using all available sparse data we randomly selected only 50 points. We applied the same inversion procedure and uniqueness conventions as when using the complete data. Results for full period/pole/shape models are listed in Table 3 and those for partial models with only P and β determination are listed in Table 4. Naturally, with less sparse data, the discrepancy between original DAMIT models and derived models is now greater. Results from Tables 3 and 4 show that if the number of sparse data points is low, it is necessary to have dense lightcurves from at least two apparitions ($N_{lc} = 4$) to derive a reliable model. For lightcurves from only one apparition ($N_{lc} = 2$), the inversion method gives incorrect results. However, the results derived from combined data with dense lightcurves from two or more apparitions are still in good agreement with the models based on full data sets.

For some asteroids (for example (2) Pallas or (21) Lutetia), we obtained formally unique solutions (according our definition) when using reduced amount of sparse data, whereas there was no unique solution for any test data set when using complete sparse data.

3.3. Tests – conclusions

The performed tests showed that if a typical combined data set (around one hundred sparse points observed over five years and a few lightcurves from two or more apparitions) leads to a unique

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[7] The well-known $\lambda \pm 180°$ ambiguity in the spin axis direction is inevitable for disk-integrated measurements in the plane of the ecliptic (Kaasalainen & Lamberg 2006; Kaasalainen & Ďurech 2007).

Table 5. Spin state solutions for models derived from combined data sets. For each asteroid, the table lists the ecliptic longitude λ_1 and latitude β_1 of the pole direction (with a possible mirror solution λ_2, β_2), the rotation period P, the number of sparse data points N_{sp}, the number of dense lightcurves N_{lc} observed during N_{app} apparitions, and a note on the source of dense lightcurves (UAPC – Uppsala Asteroid Photometric Catalogue, PDO – Palmer Divide Observatory).

Asteroid	λ_1 [deg]	β_1 [deg]	λ_2 [deg]	β_2 [deg]	P [h]	N_{sp}	N_{lc}	N_{app}	Note
5 Astraea	126	40	310	44	16.80061	153	24	7	UAPC
17 Thetis	55	10	236	20	12.26603	221	52	8	UAPC
30 Urania	107	23	284	20	13.68717	106	11	3	UAPC
34 Circe	94	35	275	51	12.17458	114	16	5	UAPC + new obs.
80 Sappho	6	−16	194	−26	14.03087	125	12	4	UAPC
82 Alkmene	164	−34	351	−39	13.00078	129	11	1	UAPC
132 Aethra	337	70			5.168274	174	4	2	UAPC
146 Lucina	139	−14	305	−41	18.55397	125	22	4	UAPC
152 Atala	199	62	347	47	6.24472	101	2	1	UAPC
182 Elsa	72	−84	224	−82	80.166	118	9	2	UAPC
184 Dejopeja	14	51	196	50	6.441119	133	4	2	UAPC
278 Paulina	123	45	311	28	6.49387	154	3	1	UAPC
360 Carlova	129	65	350	55	6.189596	143	6	2	UAPC
416 Vaticana	291	12			5.371597	131	24	2	UAPC + new obs.
484 Pittsburghia	69	47			10.64976	100	2	1	UAPC
516 Amherstia	80	53	253	22	7.48430	125	5	2	UAPC
614 Pia	165	32	354	45	4.57870	86	2	1	UAPC
628 Christine	24	−61	209	−34	16.17293	117	6	1	PDO
714 Ulula	40	−4	225	−13	6.99838	156	4	1	UAPC
770 Bali	68	44	256	40	5.81894	87	2	1	UAPC
849 Ara	17	−10	213	−33	4.116391	133	5	1	UAPC
915 Cosette	185	50	348	55	4.469741	88	1	1	UAPC
1012 Sarema	51	64	254	53	10.30708	74	2	1	UAPC
1088 Mitaka	115	−46	278	−72	3.035377	74	1	1	UAPC

solution of the inverse problem, then this solution is very close to the solution based on the full data set of dense lightcurves. The period is determined correctly within the expected error. The uncertainty in the pole direction is 20–25° at most with a typical value ~10–20°. Derived shape models are coarse and represent only basic characteristics of shape. As an example, there are three models of asteroid (130) Elektra in Fig. 1. The model derived from the full set of dense lightcurves (49 lightcurves from 8 apparitions) is compared with the model derived from the full sparse data (113 points) and one dense lightcurve and with the model derived from sparse data reduced to 50 points and four dense lightcurves from two apparitions. The sparse+dense models look rougher because their shapes are represented by spherical harmonics series, whereas the full-set model looks smoother because it has higher resolution and is represented directly by areas of surface facets (see Kaasalainen & Torppa 2001). Due to limited geometry and a small number of data points, sparse photometry usually does not allow us to reconstruct a higher-resolution model. However, the global shape features of all models are similar and the difference between pole directions of the full-set model and both sparse-data models is only 7°.

4. Results

We combined lightcurves from UAPC and PDO with sparse data from USNO, obtaining combined data sets for about 470 asteroids (350 from UAPC and 120 from PDO). We removed clear outliers from the sparse data and then applied the lightcurve inversion method to these combined data sets as described in Kaasalainen (2004); Ďurech et al. (2005, 2007). Because we
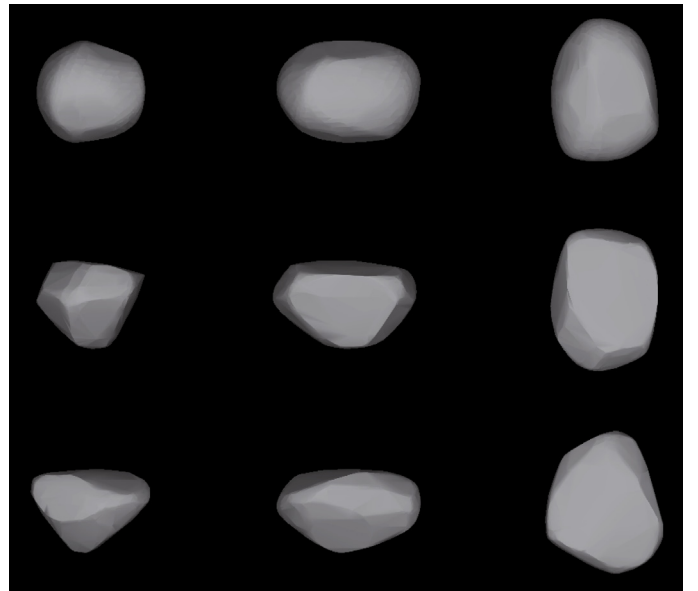
Fig. 1. *Top*: a convex shape model of (130) Elektra based on 49 standard lightcurves from 8 apparitions. *Middle*: a shape model based on sparse data (113 points) and one standard lightcurve. *Bottom*: a shape model based on reduced sparse data (50 points) and four standard lightcurves from two apparitions. Models are shown from the equator (*left, center*) and pole-on (*right*). The difference in pole directions between the full-set model and those derived using combined data is 7° in both cases.

had a priori information about the rotation periods of individual asteroids (taken from the Minor Planet Lightcurve Parameters

Table 6. Partial spin state solutions for asteroids with an ambiguous pole solution. For each asteroid, the table lists the mean ecliptic latitude β of the pole direction and its dispersion Δ. The other parameters are the same as in Table 5.

| Asteroid	β [deg]	Δ [deg]	P [h]	N_{sp}	N_{lc}	N_{app}	Note
28 Bellona	−6	17	15.70785	130	8	3	UAPC
119 Althaea	−62	8	11.46512	102	4	2	UAPC
136 Austria	63	21	11.49660	130	4	1	UAPC
312 Pierretta	−52	12	10.20768	118	4	1	UAPC
355 Gabriella	69	7	4.82899	128	4	1	UAPC
390 Alma	−64	18	3.74116	109	2	1	PDO
394 Arduina	−71	16	16.6218	108	8	1	UAPC
540 Rosamunde	57	9	9.34778	96	3	1	UAPC
544 Jetta	−66	19	7.74526	104	3	1	PDO
550 Senta	−64	18	20.5727	103	9	1	UAPC
636 Erika	−52	14	14.60755	135	6	1	UAPC
825 Tanina	54	9	6.93981	114	2	1	UAPC
966 Muschi	−57	17	5.35531	111	3	1	PDO
1089 Tama	−21	20	16.4655	61	1	1	UAPC
1188 Gothlandia	−52	15	3.491820	98	2	1	UAPC
1207 Ostenia	−57	13	9.07129	87	2	1	UAPC
1270 Datura	59	12	3.358100	79	2	1	UAPC
1514 Ricouxa	71	16	10.42466	68	3	1	UAPC

database) we could restrict the period search interval to a relatively narrow range of 1, 3, and 10% around the reported value of period for reliability codes 4, 3, and 2, respectively[8]. For most asteroids in our sample, the data were insufficient to derive a unique model – usually there were more than one equally good solutions for the rotation period or there were multiple independent pole solutions for one period. We derived 24 models (listed in Table 5) that were unique according to the definition from Sect. 3: a unique solution gives at least 10% lower χ^2 than all others outside the range of parameter errors. The weight of sparse data was one third of that of dense lightcurves. However, the models we present are not sensitive to the particular choice of weighting. The uncertainty in the rotation period depends on the length of observation interval and corresponds to the order of the last decimal place of period P in Tables 5 and 6. The uncertainty in the pole direction depends on the number of lightcurves and sparse data points and, according to tests, is usually ∼10–20°.

We also derived 18 partial models (listed in Table 6) with a uniquely determined rotation period and a rough estimation of the pole ecliptic latitude β. Each value of β given in Table 6 is a mean of the pole ecliptic latitudes for different models if their dispersion was not larger than 50°. The parameter Δ in Table 6 gives the estimated error of β, $\Delta = |\beta_{max} - \beta_{min}|/2$, where β_{max} and β_{min} are extremal values of β for individual solutions.

4.1. Selected objects

(184) Dejopeja. A model of Dejopeja was recently derived by Marciniak et al. (2007). Their pole solutions (18°, +54°) and (201°, +52°) are only a few degrees away from our solutions based on only four lightcurves from two apparitions and 133 sparse data points.

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[8] The meaning of reliability codes is explained in the footnote in Sect. 3.
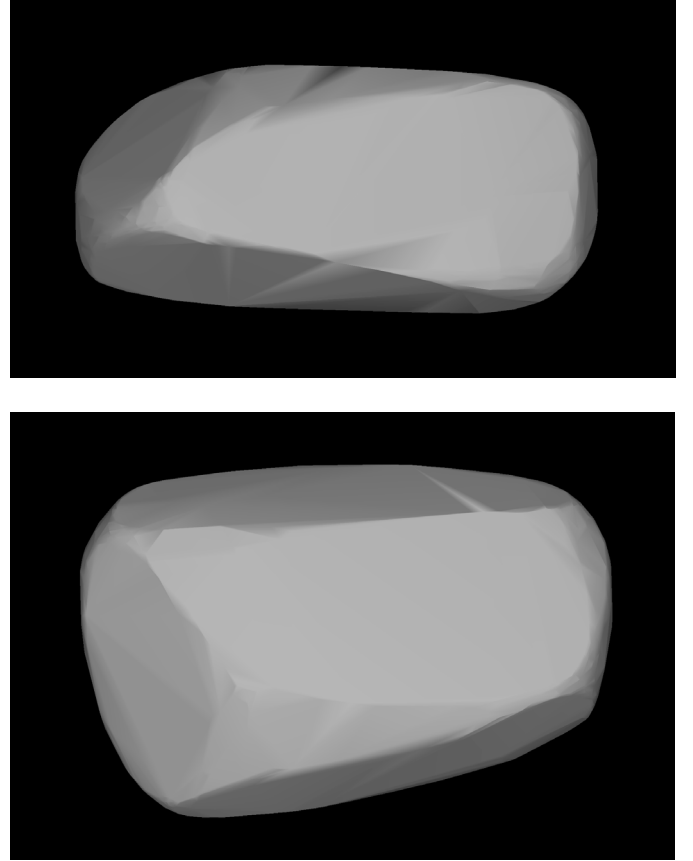
Fig. 2. A possible convex shape model of (1089) Tama shown from the equator (*top*) and pole-on (*bottom*). The flat surfaces indicate possible binarity. The model was derived from only 61 sparse data points and one dense lightcurve.

(1089) Tama. The binary nature of this asteroid was revealed by Behrend et al. (2004). We were not able to obtain a unique model. In Fig. 2, we present one possible solution with the pole direction (181°, −8°). Although the solution is derived from only one flat dense lightcurve and 61 sparse points, the shape model indicates that the asteroid could be a binary. In general, convex shape models cannot describe binary objects. However, flat shape models with rectangular pole-on silhouettes are typical results when inverting lightcurves of synchronous binaries (Ďurech & Kaasalainen 2003). According to Table 4, results derived for Tama should be taken with care due to the small number of lightcurves and sparse points. However, because Tama's brightness variations have a large amplitude of ∼1 mag (Behrend et al. 2006), the lightcurve signal is so strong that even a small number of noisy measurements yields a unique and robust solution for the rotation period.

5. Conclusions

After the theoretical work of Kaasalainen (2004) and simulations made by Ďurech et al. (2005, 2007), this paper is the first example of using real sparse data for asteroid modelling. The results we obtained clearly show that even noisy sparse data carry information about asteroid brightness variations caused by rotation and shape, and that they can be used as complementary data to ordinary lightcurves. Apart from asteroids (34) Circe and (416) Vaticana, all new models were derived just by "data mining" of

UAPC, PDO, and USNO catalogues. Although models derived from a limited number of dense lightcurves and noisy sparse data are not as accurate as standard models from multiple-apparition photometry, 24 new models significantly increase the number of asteroids with known spin states and shapes (DAMIT contains about 80 asteroid models). The 18 partial models with unknown λ but roughly estimated β can be used for statistical studies. All derived models are available through the DAMIT web page and will be updated when new observations are available. The source codes for lightcurve inversion software are also available on the DAMIT web page. We will describe the software packages in detail in a forthcoming paper.

In the near future, sparse photometric observations from sky surveys will drastically enlarge the number of asteroid models to 10^4 or even 10^5, allowing us to obtain comprehensive maps of various asteroid populations. Dense lightcurves will be important in this study. In addition to constructing the subset of best-defined models from sky surveys, they facilitate quality and reliability checks for the photometric data and the modelling pipelines of the surveys. They also serve as follow-up data for objects labelled especially interesting in the sparse photometry pipelines. Thus, dedicated dense lightcurve observation campaigns for a large number of objects will be more important than ever.

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