A&A 624, A12 (2019)
https://doi.org/10.1051/0004-6361/201834833

Astronomy & Astrophysics

Comment on "Hubble flow variations as a test for inhomogeneous cosmology"

David L. Wiltshire

School of Physical and Chemical Sciences, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand
e-mail: David.Wiltshire@canterbury.ac.nz

Received 12 December 2018 / Accepted 12 February 2019

ABSTRACT

Saulder et al. (2019, A&A, 622, A83) have performed a novel observational test of the local expansion of the Universe for the standard cosmology as compared to an alternative model with differential cosmic expansion. Their analysis employs mock galaxy samples from the Millennium Simulation, a Newtonian N–body simulation on a ΛCDM background. For the differential expansion case the simulation has been deformed in an attempt to incorporate features of a particular inhomogeneous cosmology: the timescape model. It is shown that key geometrical features of the timescape cosmology have been omitted in this rescaling. Consequently, the differential expansion model tested by Saulder et al. (2019) cannot be considered to approximate the timescape cosmology.

Key words. cosmology: observations – dark energy – large-scale structure of Universe

1. Introduction

The standard Lambda cold dark matter (ΛCDM) cosmology is built on the assumption that average cosmic expansion exactly follows that of a Friedmann–Lemaître–Robertson–Walker (FLRW) model, and that all deviations from uniform expansion are described by peculiar velocities, which can be expressed exactly in terms of local Lorentz boosts about the FLRW background. However, this is not true of general inhomogeneous cosmological solutions in general relativity, nor in any theory that incorporates key principles of general relativity.

Any observational test of differential cosmic expansion is therefore an important probe of the foundations of the standard cosmology. Given the high degree of isotropy of the cosmic microwave background (CMB), a notion of average isotropic expansion does apply on the large scale (though not necessarily given by the FLRW model). Tests of differential expansion must therefore be performed on scales comparable to that over which an average isotopic expansion is seen to emerge, namely scales of order at least 70–120 h^{-1} Mpc (Hogg et al. 2005; Scrimgeour et al. 2012), and ideally extending to a few times this scale.

Tests of differential cosmic expansion on such scales rely on very large catalogues of galaxy, group and cluster distances and redshifts, which are noisy and are subject to numerous observational biases which must be accounted for. Furthermore, any tests are ideally performed in a model independent manner, which also requires removing assumptions of the FLRW model which are often taken for granted in many analyses. To date, such a model independent test has been performed for full sky spherical averages of local expansion (Wiltshire et al. 2013; McKay & Wiltshire 2016), using the COMPOSITE (Watkins et al. 2009; Feldman et al. 2010) and Cosmicflows-II (Courtois & Tully 2012) catalogues. It was found with very strong Bayesian evidence that the spherically averaged expansion is significantly more uniform in the rest frame of the Local Group (LG) of galaxies than in the standard CMB rest frame (Wiltshire et al. 2013). However, while this may at first seem at odds with the expectations of the standard cosmology, it was subsequently shown by Kraljic & Sarkar (2016) that the result is consistent with Newtonian N-body simulations in the ΛCDM framework, given a suitably large bulk flow.

In a new paper, Saulder et al. (2019) rigorously perform a new type of test of differential expansion that they have previously proposed (Saulder et al. 2012). They consider line-of-sight averages that account for intervening structures on each line of sight, and possible effects on the variation of expansion. This is a considerably more ambitious test than the previous tests involving spherical averages, as it requires detailed knowledge of the intervening structures on any line of sight sampled. This can compound any problems relating to observational and statistical biases.

Saulder et al. (2019) analyse fundamental plane distances (Saulder et al. 2013, 2015) which they combine with information from the SDSS (Alam et al. 2015) and 2MRS (Huchra et al. 2012) surveys to create a catalogue of structures in the local Universe covering some 22.7% of the northern hemisphere sky. Setting aside possible systematic uncertainties arising from incomplete sky coverage, which Saulder et al. (2019) discuss, then the test that they propose based on determining the fraction of "finite infinity regions" (Ellis et al. 1984; Wiltshire 2007a) along individual lines of sight is a perfectly reasonable one, given robust model-independent estimates of the masses of all galaxies along the lines of sight.

In a magnitude-limited survey, and with large intrinsic scatter in the data, model-independent estimates of the masses of all galaxies close to the line of sight are impossible. Consequently, Saulder et al. (2019) choose to estimate both the masses and systematic uncertainties in the case of the ΛCDM model in a combined analysis that includes mock galaxy catalogues from the Millennium Simulation (Springel et al. 2005). While certainly justified in the case of the ΛCDM model, any rescaling of the simulation to attempt to mimic non-FLRW inhomogeneous expansion has inherent problems. In addition to various systematic issues discussed by Saulder et al. (2019), I will point out a further geometrical issue that they have not considered.

2. Key geometrical features of the timescape cosmology

The timescape model (Wiltshire 2007a,b, 2009) is a phenomenological cosmology model without dark energy, which provides one possible interpretation of the Buchert (2000, 2001) averaging scheme for general inhomogeneous cosmological solutions in general relativity. In this framework Einstein's equations apply exactly on small scales on which the fluid approximation for the average energy momentum tensor holds. However, average evolution on larger scales need not follow an exact solution of Einstein's equations. In particular, it need not coincide with a FLRW model with constant spatial curvature on large scales. A dynamical coupling of matter and geometry on small scales which allows spatial curvature to vary is a natural feature of general relativity. The requirement that spatial curvature remains constant on arbitrarily large scales of cosmological averaging is *not* a natural consequence of any principles of general relativity. Rather the FLRW models are historically the best known and tested means of imposing average spatial homogeneity and isotropy on the largest scales, to be consistent with observations, albeit with the introduction of dark energy.

Since generic inhomogeneous cosmologies can exhibit arbitrarily large differential expansion, any proposal to describe average cosmic evolution which differs substantially from the FLRW model must explain why average cosmic expansion nonetheless appears to be so close to homogeneous and isotropic. An interpretative framework is also required for the Buchert averaging scheme, since it deals with statistical volume averages and does not automatically incorporate a means to relate local observables to the statistical quantities.

In the timescape model both of these matters are dealt with by revisiting Einstein's strong equivalence principle, and extending it to general averages of the cosmological Einstein equations (Wiltshire 2008). In the presence of gradients of spatial curvature between expanding regions of vastly different densities, the regional Hubble parameter related to the quasilocal[1] expansion is calibrated in terms of regional rulers and clocks. But the relative calibration can vary from region to region.

The observation of average spatial homogeneity is then accounted for differently. As a consequence of the cosmological equivalence principle (Wiltshire 2008) it is recognized that expansion appears to be uniform because the actual quasilocal expansion *is* uniform in terms of a canonical choice of regional rulers and clocks that varies from region to region. In a universe which grows to be dominated in volume by (negatively curved) voids at late epochs, there is a systematic drift between the volume-average rulers and clocks (that best describe average cosmic evolution) and the rulers and clocks of ideal observers in overdense regions where the mass of the universe is mostly concentrated. Implementing this requires care.

In the "two phase" model that has been studied to date (Wiltshire 2007a,b, 2009; Duley et al. 2013) the *average volume*, $\mathcal{V} = \mathcal{V}_i \bar{a}^3$, expands as a disjoint union of spatially flat "walls" and negatively curved voids. The walls are formally a union of the "finite infinity regions" (Wiltshire 2007a), which are the compact boundaries enclosing all gravitationally bound structures within which the density averages to the timescape model critical density. The volume-average scale factor, $\bar{a}$, is related to the regional scale factors $a_{\rm w}$ and $a_{\rm v}$ of the walls and voids respectively by

$$\bar{a}^3 = f_{\rm wi} a_{\rm w}^3 + f_{\rm vi} {a_{\rm v}}^3 \tag{1}$$

where $f_{\rm wi}$ and $f_{\rm vi} = 1 - f_{\rm wi}$ represent the fraction of the initial volume, $\mathcal{V}_i$, in wall and void regions respectively in the very early universe when $f_{\rm vi} \ll 1$. One may rewrite Eq. (1) as

$$f_{\rm v}(t) + f_{\rm w}(t) = 1, \tag{2}$$

where $f_{\rm w}(t) = f_{\rm wi} a_{\rm w}^3/\bar{a}^3$ is the wall volume fraction and $f_{\rm v}(t) = f_{\rm vi} a_{\rm v}^3/\bar{a}^3$ is the void volume fraction. Taking a derivative of Eq. (1) with respect to the Buchert time parameter, t, gives

$$\bar{H} \equiv \frac{\partial_t \bar{a}}{\bar{a}} = f_{\rm w} H_{\rm w} + f_{\rm v} H_{\rm v}, \tag{3}$$

where $H_{\rm w} \equiv (\partial_t a_{\rm w})/a_{\rm w}$ and $H_{\rm v} \equiv (\partial_t a_{\rm v})/a_{\rm v}$. This expresses the relation between the "bare Hubble parameter", $\bar{H}$, and effective Hubble parameters of the walls and voids respectively as determined by volume-average clocks. This parameterization allows the Buchert evolution equation (Buchert 2000) to be written in a form reminiscent of the Friedmann equation,

$$\bar{\Omega}_{\rm M} + \bar{\Omega}_{\rm R} + \bar{\Omega}_{\rm k} + \bar{\Omega}_{Q} = 1, \tag{4}$$

where $\bar{\Omega}_{\rm M} \equiv \bar{\rho}_{\rm M0}\bar{a}_0^3/(\bar{a}^3\bar{\rho}_{\rm cr})$, $\bar{\Omega}_{\rm R} \equiv \bar{\rho}_{\rm R0}\bar{a}_0^4/(\bar{a}^4\bar{\rho}_{\rm cr})$, $\bar{\Omega}_{\rm k} \equiv 3\alpha^2 f_{\rm v}^{1/3}/(8\pi G\bar{a}^2\bar{\rho}_{\rm cr})$, $\bar{\Omega}_{Q} \equiv -(\partial_t f_{\rm v})^2/[24\pi G f_{\rm v}(1-f_{\rm v})\bar{\rho}_{\rm cr}]$,

$$\bar{\rho}_{\rm cr} \equiv 3\bar{H}^2/(8\pi G) \tag{5}$$

is the timescape model critical density, $\bar{\rho}_{\rm M0}$ and $\bar{\rho}_{\rm R0}$ are the present epoch volume-average matter and radiation densities, $\alpha^2 \equiv -k_{\rm v} f_{\rm vi}^{2/3} c^2$, and $k_{\rm v} < 0$ is the curvature scale of the voids.

While the matter and radiation density parameters, $\bar{\Omega}_{\rm M}$ and $\bar{\Omega}_{\rm R}$, scale with the average volume, $\bar{a}^3$, in a similar manner to the FLRW model, the spatial curvature fraction, $\bar{\Omega}_{\rm k}$, is very different since its time-variation depends not only on the average volume, but also on the fraction of that volume occupied by voids, $f_{\rm v}$. Finally the kinematic backreaction term, $\bar{\Omega}_{Q}$, is entirely absent in the FLRW model. Equation (4) is supplemented by an additional equation for the second derivative of $f_{\rm v}$ (Wiltshire 2007a,b, 2009).

Equations (4), (5) refer to averaged geometrical quantities in terms of a statistically average clock, but apart from the two phase approximation do not place any restriction on the Buchert scheme. The timescape model implements the further restriction of the uniform quasilocal Hubble flow condition as follows. Radial light rays within finite infinity regions where the metric,

$$ds_{\rm fi}^2 = -c^2 d\tau^2 + {a_{\rm w}}^2(\tau)\left[d{\eta_{\rm w}}^2 + {\eta_{\rm w}}^2 d{\Omega_2}^2\right], \tag{6}$$

is regionally spatial flat, are matched conformally to radial light rays in a general spherically symmetric metric fit to a solution to the Buchert equations. This results in an effective *dressed* metric on radial lines of sight,

$$ds^2 = -c^2 d\tau^2 + a^2(\tau)\left[d\bar{\eta}^2 + {r_{\rm w}}^2(\bar{\eta},\tau)\, d{\Omega_2}^2\right], \tag{7}$$

where $a \equiv \bar{\gamma}^{-1}\bar{a}$,

$$r_{\rm w} = \bar{\gamma}_0 (1-f_{\rm v})^{1/3} \int_t^{t_0} \frac{c\, dt'}{\bar{\gamma}(t')(1-f_{\rm v}(t'))^{1/3}\bar{a}(t')}, \tag{8}$$

$\bar{\gamma}_0 = \bar{\gamma}(t_0)$, and

$$\bar{\gamma} \equiv \frac{d\tau}{dt}, \tag{9}$$

is the phenomenological lapse function which gives the difference of the proper time parameter, τ, of ideal observers in finite

[1] The word "local" as typically used in phrases such as "the local Universe" is ambiguous as it implies a choice of scale. In general relativity, "local" strictly means the neighbourhood of a point over which gravity can be neglected – scales much smaller than galaxies. As soon as one deals with larger regions in which gravity cannot be neglected then another terminology – quasilocal – is required.

infinity regions (in bound structures like ourselves) from the statistical volume-average time, t, that best encodes average cosmic evolution.

There are two important geometrical issues to note. Firstly, the effective dressed metric Eq. (7) is not spatially of constant curvature, and thus cannot be obtained by a spatial rescaling of a FLRW metric. Secondly, the dressed Hubble parameter

$$H \equiv \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau} = \frac{1}{\bar{a}}\frac{\mathrm{d}\bar{a}}{\mathrm{d}\tau} - \frac{1}{\bar{\gamma}}\frac{\mathrm{d}\tau}{\mathrm{d}\bar{\gamma}} = \bar{\gamma}\bar{H} - \frac{\mathrm{d}\bar{\gamma}}{\mathrm{d}t}, \tag{10}$$

not only has a contribution from rate of change of the average volume, $\bar{a}^{-1}\partial_\tau\bar{a}$, but also a contribution, $-\bar{\gamma}^{-1}\partial_\tau\bar{\gamma}$, from the rate of change of the phenomenological lapse function.

As a numerical example, from a fit of the angular scale of the CMB acoustic peaks using Planck satellite data, Duley et al. (2013) find a bare Hubble constant, $\bar{H}_0 = 50.1 \pm 1.7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and a present epoch lapse $\bar{\gamma}_0 = 1.348^{+0.021}_{-0.025}$. The corresponding dressed Hubble constant is $H_0 = 61.7 \pm 3.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, which can be understood as being equal to the fractional τ-rate of change of the volume-average scale factor, $\bar{H}_{\tau,0} \equiv \bar{a}_0^{-1}\partial_\tau\bar{a}\big|_0 = \bar{\gamma}_0\bar{H}_0 = 67.5^{+3.4}_{-3.5}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, as further reduced by $-\bar{\gamma}_0^{-1}\partial_\tau\bar{\gamma}\big|_0 \sim -5.8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. We note that the nonreduced value $\bar{H}_{\tau,0}$ matches the value of the Hubble constant obtained for the FLRW model from the same data (Ade et al. 2014), albeit with larger uncertainties since only the angular scale of the acoustic peaks is fitted. This is precisely what should be expected since the data fit compares the same physical scales between last scattering and the present epoch: if one ascribes this change solely to the rate of change of an average volume by the same clock then there is a unique result.

3. Systematic problem of the Saulder et al. (2019) rescaling

To create mock samples for calibrating their differential expansion model, Saulder et al. (2016, 2019) determine finite infinity radii from the "*millimil*" run of the Millennium simulation as

$$R_\mathrm{f} = \left(\frac{3M_\mathrm{tot}}{4\pi\rho_\mathrm{cr}f}\right)^{1/3}, \tag{11}$$

where M_tot is the mass within a simulated finite infinity region (Saulder et al. 2016, Sect. 3.5), ρ_cr is the critical density of the simulation, and f is a factor that modifies the critical density according to

$$f = \left(\frac{\bar{H}_0}{H_0}\right)^2 \simeq \left[\frac{2(2+f_{\mathrm{v}0})}{4{f_{\mathrm{v}0}}^2 + f_{\mathrm{v}0} + 4}\right]^2, \tag{12}$$

where $\bar{H}_0 = \bar{H}(t_0)$ and $H_0 = H(t_0)$ are respectively the bare and dressed Hubble constants for the timescape model as above, $f_{\mathrm{v}0} = f_\mathrm{v}(t_0)$, and in the last equality we have used the exact tracking limit solution (Wiltshire 2007b, 2009).

Using the above definitions, Saulder et al. (2019) empirically define average fractions of lines of sight in finite infinity regions from the mock catalogues according to

$$f_\mathrm{fi}\widetilde{H}_{0,\mathrm{w}} + (1 - f_\mathrm{fi})\widetilde{H}_{0,\mathrm{v}} = \widetilde{H}_{0,\mathrm{av}}, \tag{13}$$

assuming $\widetilde{H}_{0,\mathrm{av}}$ to coincide with the dressed Hubble constant, H_0. We introduce a tilde on $\widetilde{H}_{0,\mathrm{w}}$ and $\widetilde{H}_{0,\mathrm{v}}$ to distinguish these empirical quantities from the present epoch values of H_w and H_v as given in Eq. (3). Equation (3) is a volume average using the time parameter, t, whereas Eq. (13) invokes an average over 1-dimensional lines of sight, with expansion referred to our own time parameter, τ. By the uniform quasilocal Hubble flow condition, the regionally measured Hubble constant within spatially flat finite infinity regions would coincide with the bare Hubble constant, i.e., $\widetilde{H}_{0,\mathrm{w}} = \bar{H}_0 = 50.1 \pm 1.7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. However, Eq. (10) which defines the dressed Hubble parameter is not linearly related to any void fraction. Thus the quantity $\widetilde{H}_{0,\mathrm{v}}$ and the relation Eq. (13) have no obvious counterparts in the timescape model.

There is, furthermore, a geometrical problem with the volume-based assumption Eq. (11) that has gone into this construction. In flat space a sphere of radius, R, has Euclidean volume $V_\mathrm{E} = \frac{4}{3}\pi R^3$. For a negatively curved space, the volume of a sphere of the same fixed radius is *larger* than $\frac{4}{3}\pi R^3$. Equivalently, a shorter line-of-sight distance is required in a negatively curved void to obtain the same volume as the Euclidean case. Regional Hubble parameters are based on volume expansion. But the relation between line-of-sight distance and volume changes in the presence of spatial curvature gradients. Consequently, the finite infinity fraction on a line of sight to any observed galaxy must differ between the two models as a consequence of geometry.

While the bare Hubble parameter is related to the critical density $\bar{\rho}_\mathrm{cr}$ by Eq. (5), as shown by Eq. (10) the dressed Hubble parameter includes a contribution from the rate of change of the phenomenological lapse function. Thus it is not directly related to *any* scaling of the average volume as in the definition of the critical density, $\rho_\mathrm{cr} = 3{H_0}^2/(8\pi G)$, in the FLRW model.

Furthermore, since spatial curvature is not constant in the timescape model, it is impossible to simply "correct" the Saulder et al. (2016, 2019) analysis by replacing the dressed Hubble constant in Eq. (12) by the term $\bar{H}_{\tau,0} = \bar{a}_0^{-1}\partial_\tau\bar{a}\big|_0$. The underlying problem is that a density is a mass divided by a volume. For a space of constant spatial curvature the relation between radius and volume is fixed. However, in the timescape model the average spatial curvature changes with time in a way which does not scale in direct proportion to the spatial volume.

In an attempt to overcome some of the limitations of their approach, Saulder et al. (2019) introduce an additional empirical parameter, b_soft, which can be tuned to adjust the amount of differential expansion. In this way they can impose a match of the relative Hubble constant to the value $\bar{H}_0/H_0$ in the limit that the line-of-sight finite infinity fraction $f_\mathrm{fi} \to 1$, as predicted by the uniform quasilocal Hubble flow condition.

Unfortunately, even with an additional empirical scaling, a basic problem still remains. It is impossible for individual galaxies to simultaneously have identical values of both the relative line-of-sight Hubble parameter H_i/H_0 and of the line-of-sight finite infinity fraction, f_fi, as derived from a simulation by Eq. (11) in both the FLRW and timescape models[2]. In the FLRW case the relationship between the line-of-sight H_0 and the average volume of the simulation is prescribed by Euclidean geometry in any density calibration; in the timescape case it is not. For the timescape, the relationship between the dressed parameter H_0 and the average volume in the Buchert averages is highly nonlinear, since the later has negative spatial curvature. The fact that Saulder et al. (2019) plot the expectations for both ΛCDM and the differential expansion model against data with identical H_i/H_0 and f_fi values indicates that there is an inconsistency as far as a timescape approximation is concerned.

[2] This is purely a geometrical statement which holds irrespective of the fact that the finite infinity notion plays no role in the Millennium simulation, and irrespective of the fact that the relative Hubble parameters assume different values of H_0.

4. Discussion and conclusions

Nonkinematic differential cosmic expansion – i.e., a distance-redshift relation which differs from that of a FLRW model plus local Lorentz boosts – is a generic feature of inhomogeneous cosmological models (Bolejko et al. 2016). Such models include averages of the Einstein equations with backreaction, which may present viable alternatives to dark energy as a source of late epoch apparent cosmic acceleration.

Saulder et al. (2019) have conceived a novel test for differential expansion, and have undertaken a heroic effort in their detailed analysis of a large data set. The data has various systematic limitations when applied to the test in question. To overcome this, they combine the data with simulated data from the ΛCDM N-body Newtonian Millennium simulation (Springel et al. 2005). This gives a reasonable estimate of the magnitude of the Kaiser (1987) effect, which can be considered as a kinematic differential expansion (Bolejko et al. 2016). They then use a rescaled version of the simulation which accentuates the differential expansion, since equivalent simulations are unavailable in the case of the timescape model which inspired their analysis.

Although the details of differential expansion in the timescape model must differ[3] from those of the Kaiser effect in the ΛCDM model, there are no a priori grounds by which we should expect its magnitude to be greater than the Kaiser effect. Indeed, just as the ΛCDM model has a restriction on inhomogeneity – that average expansion occurs exactly in hypersurfaces of constant spatial curvature, as in a FLRW model – the timescape model also has an important simplifying restriction on inhomogeneity, namely the uniform quasilocal Hubble flow condition, Eq. (10). This restriction is geometrically very different to that of the FLRW model.

Equation (10) exactly prescribes how the time rates of change of the volume-average scale factor and phenomenological lapse must be combined to match observations when we attempt to extract a Hubble constant from distance-redshift data in the usual manner (on scales larger than the statistical homogeneity scale). Any approximation to the timescape cosmology should incorporate this restriction. As we have shown here, the Saulder et al. (2016, 2019) scaling does not.

A question remains as to whether some nonlinear deformation of the Millennium simulation could effectively approximate the restriction of Eq. (10). This is unclear. Rácz et al. (2017) have performed a simulation without dark energy, the "AvERA model", in which standard Newtonian N-body codes are evolved with the Friedmann equations on small scales, and then averaged at each time step to determine a collective volume-average scale factor in analogy to the Buchert approach. The resulting distance-redshift relation tracks very close to that of the timescape model[4]. Since it is a Newtonian N-body framework, it does allow for a direct comparison between the Millennium simulation and a phenomenological backreaction framework (Beck et al. 2018). However, since the scheme is arrived at by making empirical changes to cosmic evolution at the level of a computer code, it is unclear how these changes relate directly to physical questions associated with effective spatial curvature or simplifying physical principles for average cosmic evolution, such as those which underlie the timescape model (Wiltshire 2008). Nonetheless, it may provide a framework for considering the Saulder et al. (2019) test, and deserves further investigation.

Other tests of nonkinematic differential expansion are possible. For example, apparently anomalous features in the large angle CMB multipoles are a generic prediction, as quantified by Bolejko et al. (2016, Eqs. (2.3), (2.4)). Simple ray–tracing estimates using the Lemaître–Tolman–Bondi model show that the precision to definitively distinguish nonkinematic differential expansion from the standard expectation is not reached with present data (Dam 2016), however. That test, comparisons of the integrated Sachs–Wolfe effect (Beck et al. 2018), and the Saulder et al. (2019) test are all complementary ways for testing the possibility of nonkinematic differential expansion once substantial advances in observational precision are made.

In the case of the timescape cosmology, substantial theoretical advances are also required to make predictions with the level of detail available in N–body Newtonian simulations on the FLRW background, in order to allow a direct implementation of the proposed test of Saulder et al. (2019).

Acknowledgements. I thank Christoph Saulder and Nezihe Uzun for helpful comments.

[3] Smaller scale "fingers of God" redshift-space distortions due to velocity dispersions within gravitationally bound structures are understood in the timescape model similarly to the standard cosmology. On scales on which space is expanding, however, the conceptual framework is fundamentally different.

[4] The comoving distance-redshift relation in the AvERA model (Rácz et al. 2017, Fig. 5b) is empirically very close to that of the timescape model fitted to the same Planck data (Wiltshire et al. 2014; Fig. 8a). The timescape result lies between the ΛCDM and AvERA curves, but is closer to the AvERA result overall.

References

Ade, P. A. R., Aghanim, N., Armitage-Caplan, C., et al. 2014, A&A, 571, A16
Alam, S., Albareti, F. D., Allende Prieto, C., et al. 2015, ApJS, 219, 12
Beck, R., Csabai, I., Rácz, G., & Szapudi, I. 2018, MNRAS, 479, 3582
Bolejko, K., Nazer, M. A., & Wiltshire, D. L. 2016, J. Cosmol. Astropart. Phys., 06, 035
Buchert, T. 2000, Gen. Relativ. Grav., 32, 105
Buchert, T. 2001, Gen. Relativ. Grav., 33, 1381
Courtois, H. M., & Tully, R. B. 2012, ApJ, 749, 174
Dam, L. H. 2016, M.Sc. Thesis (Christchurch: University of Canterbury), http://hdl.handle.net/10092/13167
Duley, J. A. G., Nazer, M. A., & Wiltshire, D. L. 2013, Class. Quantum Grav., 30, 175006
Ellis, G. F. R. 1984, in General Relativity and Gravitation, eds. B. Bertotti, F. de Felice, & A. Pascolini (Dordrecht: Reidel)
Feldman, H. A., Watkins, R., & Hudson, M. J. 2010, MNRAS, 407, 2328
Hogg, D. W., Eisenstein, D. J., Blanton, M. R., et al. 2005, ApJ, 624, 54
Huchra, J. P., Macri, L. M., Masters, K. L., et al. 2012, ApJS, 199, 12
Kaiser, N. 1987, MNRAS, 227, 1
Kraljic, D., & Sarkar, S. 2016, J. Cosmol. Astropart. Phys., 10, 016
McKay, J. H., & Wiltshire, D. L. 2016, MNRAS, 457, 3285, Err. ibid. 463, 3113, DOI: 10.1093/mnras/stw2210
Rácz, G., Dobos, L., Beck, R., Szapudi, I., & Csabai, I. 2017, MNRAS, 469, L1
Saulder, C., Mieske, S., & Zeilinger, W. W. 2012, Proc. of Sci., DSU2012, 018
Saulder, C., Mieske, S., Zeilinger, W. W., & Chilingarian, I. 2013, A&A, 557, A21
Saulder, C., van den Bosch, R. C. E., & Mieske, S. 2015, A&A, 578, A134
Saulder, C., van Kampen, E., Chilingarian, I. V., Mieske, S., & Zeilinger, W. W. 2016, A&A, 596, A14
Saulder, C., Mieske, S., van Kampen, E., & Zeilinger, W. W. 2019, A&A, 622, A83
Scrimgeour, M., Davis, T., Blake, C., et al. 2012, MNRAS, 425, 116
Springel, V., White, S. D. M., Jenkins, A., et al. 2005, Nature, 435, 629
Watkins, R., Feldman, H. A., & Hudson, M. J. 2009, MNRAS, 392, 743
Wiltshire, D. L. 2007a, New J. Phys., 9, 377
Wiltshire, D. L. 2007b, Phys. Rev. Lett., 99, 251101
Wiltshire, D. L. 2008, Phys. Rev. D, 78, 084032
Wiltshire, D. L. 2009, Phys. Rev. D, 80, 123512
Wiltshire, D. L. 2014, in Cosmology and Gravitation: Proc. XVth Brazilian School, eds. M. Novello, & S. E. Perez Bergliaffa (Cambridge: Cambridge Scientific Publishers), 203
Wiltshire, D. L., Smale, P. R., Mattsson, T., & Watkins, R. 2013, Phys. Rev. D, 88, 083529