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Two birds with one stone: Simultaneous realization of Lunar Coordinate Time and lunar geoid time with a single orbital clock

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ABSTRACT

Context. There are three options for defining the lunar reference time. Option O1, using Lunar Coordinate Time, has the advantage of simplicity, while options O2 – using the lunar geoid (selenoid) time – and O3 – using an average alignment with Terrestrial Time – have the advantage of convenience for users with instruments on the lunar surface and those using Earth navigation satellite signals, respectively. Clock steering must be performed for all three options. O2 and O3 provide new scalings of spatial coordinates and mass parameters in the Solar System.

Aims. We propose a 'time-aligned orbit' in which the readings of an ideal clock in this orbit are equal to the selenoid time in O2; these readings can be converted to Lunar Coordinate Time in O1 via a known linear transformation.

Methods. We show that there exists a time-aligned orbit around the Moon with a semi-major axis of about 1.5 lunar radii that slightly depends on its inclination with respect to the equator of the Moon. We conducted a set of numerical simulations to assess to what extent a clock on these orbits could be used in O2 in a more realistic lunar environment.

Results. The proper time in our simulations de-synchronizes from the selenoid time by up to 190 ns after a year with a frequency offset of 6×10^{-15}, which is only 3.75% of the frequency difference in O2 caused by the lunar surface topography. This could be further reduced to 13 ns and 4×10^{-16} if we are able to account for the deviation of the mean orbits in our simulations from the nominal ones.

Conclusions. One can simultaneously realize and use options O1 and O2 by deploying a single clock in the time-aligned orbit. This approach is scalable to other terrestrial planets beyond the Earth–Moon system.

Key words. time

1. Introduction

The definition of a lunar reference time (LRT; IAU 2024b) has drawn much attention recently. It must satisfy the following (and potentially other) criteria (Bourgoin et al. 2026):

- C1: it should be defined as a linear function of Lunar Coordinate Time (TCL; IAU 2024a).
- C2: it should be practical and available so that all actors can refer to it. It should be based on Earth's clocks until an accurate clock can be deployed on the Moon.
- C3: it should have a clear relationship with the Coordinated Universal Time (UTC).

The first criterion leads to the following three options (Bourgoin et al. 2026):

- O1: the LRT is exactly the same as the TCL.
- O2: the LRT is a coordinate time that has the same average rate of the proper time of a clock on a given lunar geoid. We refer to this as the 'selenoid time' in this paper.
- O3: the LRT deviates from Terrestrial Time (TT) by periodic variations only.

O1 has the obvious advantage of simplicity, but it would be inconvenient for clocks on the lunar surface with a frequency accuracy better than 3×10^{-11} since such a clock would have a frequency offset with respect to the TCL at the same level (Bourgoin et al. 2026). Although O2 and O3 would be convenient for individuals using clocks on the lunar surface and Earth navigation satellite signals, respectively, their scalings of the TCL would imply the same scalings of spatial coordinates and mass parameters in the Solar System, which could cause confusion. Moreover, it is still very challenging to deploy and maintain clocks on the surface of the Moon.

In an attempt to reconcile these pros and cons, we propose a method that simultaneously realizes both O1 and O2. We show that there exists an orbit around the Moon on which the readings of an ideal clock equal the selenoid time (O2). Meanwhile, one can determine the TCL (O1) by scaling the readings of such an orbital clock with a factor related to the potential of the selenoid. We call this orbit the 'time-aligned orbit'.

In Sect. 2, we explain the underlying reasons for the existence of the time-aligned orbit. We present the properties of the time-aligned orbit in a more realistic lunar environment with a numerical simulation in Sect. 3. We conclude this work and discuss its scalability to other planets in Sect. 4.

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2. Theory

In this section we explain why a time-aligned orbit exists around the Moon and extend this concept to a more generic case. Under the Lunar Celestial Reference System (LCRS; IAU 2024a), the coordinate time (TCL) and the proper time (τ) of a clock in the vicinity of the Moon satisfy the following relation (Bourgoin et al. 2026):

$$\mathrm{TCL} - \mathrm{TCL}_0 = \tau - \tau_0 + \frac{1}{c^2}\int_{\tau_0}^{\tau}\left[U_\mathrm{M}(\boldsymbol{Y}) + \frac{\dot{\boldsymbol{Y}}^2}{2}\right]\mathrm{d}\tau + O(c^{-4}), \quad (1)$$

where τ_0 is the initial reading of the clock, TCL_0 is the TCL moment corresponding to τ_0, $\boldsymbol{Y}$ and $\dot{\boldsymbol{Y}}$ are the position and velocity vectors of the clock in the LCRS, and $U_\mathrm{M}(\boldsymbol{Y})$ represents the gravitational potential at the clock from the Moon, ignoring effects from all of the other bodies in the Solar System.

Considering the proper time (τ_s) of an ideal clock on the equator of a given selenoid with its specified potential (W_{M0}), Eq. (1) leads to (Nelson 2011)

$$\mathrm{TCL} = (1 + L_\mathrm{L})(\tau_\mathrm{s} - \tau_{\mathrm{s0}}) + \mathrm{TCL}_{\mathrm{s0}} + O(c^{-4}) \quad (2)$$

with

$$L_\mathrm{L} \equiv c^{-2}W_{\mathrm{M0}} \approx \frac{GM_\mathrm{M}}{c^2 R_\mathrm{M}}\left(1 + \frac{1}{2}J_2^\mathrm{M} + \frac{1}{2}\eta_\mathrm{M}\right), \quad (3)$$

where M_M, R_M, and J_2^M are the mass, mean equatorial radius, and dynamical form factor of the Moon, and we neglect the higher-order spherical harmonic components of U_M. The parameter η_M represents the ratio

$$\eta_\mathrm{M} = \frac{\dot{\boldsymbol{Y}}_\mathrm{M}^2}{\mathcal{V}_\mathrm{M}^2}, \quad (4)$$

where $\dot{\boldsymbol{Y}}_\mathrm{M}$ is the rotational velocity at the location of the clock on the surface of the Moon, and $\mathcal{V}_\mathrm{M} = \sqrt{GM_\mathrm{M}/R_\mathrm{M}}$ is the first cosmic velocity of the Moon.

For the proper time (τ_p) of an ideal clock in a mean circular orbit ($\bar{e}_\mathrm{p} = 0$) around the Moon, Eq. (1) gives (Kouba 2004; Formichella et al. 2021)

$$\mathrm{TCL} = (1 + L_\mathrm{P})(\tau_\mathrm{p} - \tau_{\mathrm{p0}}) + \mathrm{TCL}_{\mathrm{p0}} + O(c^{-4}) \quad (5)$$

with

$$L_\mathrm{P} \equiv \frac{3}{2}\frac{GM_\mathrm{M}}{c^2\bar{a}_\mathrm{p}}\left[1 + \frac{7}{3}J_2^\mathrm{M}\frac{R_\mathrm{M}^2}{\bar{a}_\mathrm{p}^2}\left(1 - \frac{3}{2}\sin^2\bar{i}_\mathrm{p}\right)\right], \quad (6)$$

where $\bar{a}_\mathrm{p}$, $\bar{e}_\mathrm{p}$, and $\bar{i}_\mathrm{p}$ are the mean semi-major axis, eccentricity, and inclination of the orbit with respect to the equator of the Moon, respectively, and we neglect the higher-order spherical harmonic components of U_M. Although the contribution of J_2 in Eq. (6) is well known, sign errors and a missing factor are found in the literature, as detailed by Formichella et al. (2021).

If we choose TCL for the coordinate simultaneity of these two clocks' proper times (Eqs. (2) and (5)), we have

$$\tau_\mathrm{s} = \frac{1 + L_\mathrm{P}}{1 + L_\mathrm{L}}\tau_\mathrm{p} + \tau_{\mathrm{s0}} - \frac{1 + L_\mathrm{P}}{1 + L_\mathrm{L}}\tau_{\mathrm{p0}} + \frac{\mathrm{TCL}_{\mathrm{p0}} - \mathrm{TCL}_{\mathrm{s0}}}{1 + L_\mathrm{L}}. \quad (7)$$

After adjusting the initial constants in the above equation, we can align the orbital proper time (τ_p) with the selenoid time, i.e. $\tau_\mathrm{s} = \tau_\mathrm{p}$, as long as $L_\mathrm{P} = L_\mathrm{L}$. This leads to the specific mean semi-major axis

$$\bar{a}_\mathrm{p} = \frac{3}{2}\frac{GM_\mathrm{M}}{c^2 L_\mathrm{L}}\left[1 + \frac{28}{27}J_2^\mathrm{M}L_\mathrm{L}^2\left(\frac{GM_\mathrm{M}}{c^2 R_\mathrm{M}}\right)^{-2}\left(1 - \frac{3}{2}\sin^2\bar{i}_\mathrm{p}\right)\right], \quad (8)$$

where we neglect the non-linear effect of J_2^M. We call such an orbit the 'time-aligned orbit'. It has two distinctive properties: (i) the proper time of an ideal clock in the time-aligned orbit can naturally equal the selenoid time, which has the same average rate of the proper time of a clock on the selenoid; and (ii) the TCL can easily be found by scaling the proper time of an ideal clock in the time-aligned orbit with a known factor related the potential of the selenoid, i.e. $\mathrm{TCL} = (1 + L_\mathrm{L})\tau_\mathrm{p}$ + constant.

We can drop the J_2^M term in Eq. (8) when specific inclination $3\sin^2\bar{i}_\mathrm{p} = 2$, and can thus determine the mean semi-major axis of the time-aligned orbit around the Moon using the following equation:

$$\bar{a}_\mathrm{p} = \frac{3}{2}\frac{GM_\mathrm{M}}{c^2 L_\mathrm{L}} = 2605.9\ \mathrm{km}. \quad (9)$$

Here we adopted $L_\mathrm{L} = 3.14027 \times 10^{-11}$ (Ardalan & Karimi 2014). Then, using Eqs. (3) and (8), we can write

$$\bar{a}_\mathrm{p} \approx \frac{3}{2}R_\mathrm{M}\left[1 + J_2^\mathrm{M}\left(\frac{29}{54} - \frac{14}{9}\sin^2\bar{i}_\mathrm{p}\right) - \frac{1}{2}\eta_\mathrm{M}\right] \approx 1.5\,R_\mathrm{M}, \quad (10)$$

since $J_2^\mathrm{M} \ll 1$ and $\eta_\mathrm{M} \ll 1$ for the Moon. This provides us with the more helpful insight that if the rotational surface speed of a nearly spherical body is much lower than its first cosmic speed, then it has a time-aligned orbit above its surface with a height of about a half of the body's radius.

With the help of the International Astronomical Union resolutions (Soffel et al. 2003; IAU 2024a), we can trace the proper time (τ_p) of an ideal clock in the time-aligned orbit back to UTC via

$$\mathrm{UTC} = (1 - k)\,\tau_\mathrm{p} + P + \mathrm{const.} \quad (11)$$

Here the clock can have a significant frequency offset (k) of about $6.5 \times 10^{-10} = 56\,\mu\mathrm{s\,d}^{-1}$ with respect to the UTC (Kopeikin & Kaplan 2024; Turyshev et al. 2025), P includes all of the periodic variations with amplitudes of up 1.6 ms, and the constant is a combination of the initial reading of the clock and other defining constants.

3. Simulation

To understand whether and to what extent a clock on the predicted time-aligned orbit could be used to realize the selenoid time in a realistic lunar environment, we carried out a set of numerical simulations. In these simulations, we included the point-mass gravitational effects of the Moon, Sun, and all planets and the higher-order spherical harmonics of the Moon. Effects of the Sun and planets are included through the tidal potential in the TCL-τ equation (Eq. (1)) and through the gravitational perturbations in the equations of motion of the orbital clock. Table 1 compares the model we used for theoretical analysis in Sect. 2 with that used in our numerical simulations.

We chose four different orbits, with initial inclinations of $i_{\mathrm{p},0} = \{0, 25°, 54.74°, 85°\}$ and initial semi-major axes ($a_{\mathrm{p},0}$) calculated based on Eq. (8). We propagated the trajectory of each

Table 1. Comparison of the models for theoretical analysis and numerical simulations.

	Theoretical analysis	Numerical simulation
Point-mass	Moon	Moon Sun 8 planets
Nonspherical Effects	J_2 term of the Moon	Up to the 100th-degree harmonics of the Moon

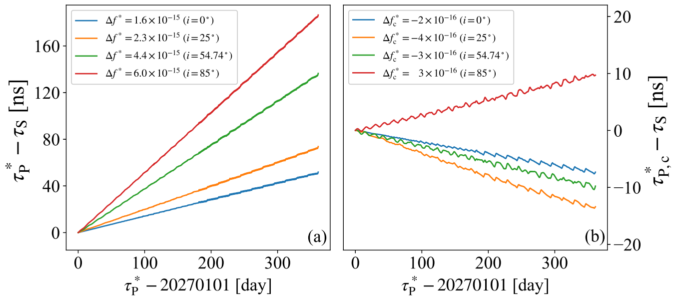

Fig. 1. Left panel: de-synchronizations ($\Delta^* = \tau^*_\mathrm{p} - \tau_s$) and frequency offsets (Δf^*) of four numerically simulated time-aligned orbits with different initial inclinations. Right panel: corrected $\Delta^*_\mathrm{c} = \tau^*_\mathrm{p,c} - \tau_s$ and Δf^*_c for the same four simulated orbits by accounting for the deviation of the mean orbital elements in our simulations from the ones required by the time-aligned orbits.

orbit for a year. By using Eq. (1), we can calculate the proper time (τ^*_p) on each simulated orbit from the TCL. As such, we can obtain the de-synchronization,

$$\Delta^* = \tau^*_\mathrm{p} - \tau_\mathrm{s}, \tag{12}$$

which indicates how well a clock on the time-aligned orbit can be used to realize the selenoid time. We can derive its frequency offset (Δf^*) to determine the drift rate between them using

$$\Delta f^* = \left\langle \frac{\mathrm{d}\Delta^*}{\mathrm{d}\tau^*_\mathrm{p}} \right\rangle, \tag{13}$$

where $\langle\cdot\rangle$ means the average over a long-term time span.

Figure 1a shows that the de-synchronization (Δ^*) grows with the initial inclination, from $\Delta^* = 50$ ns for $i_{\mathrm{p},0} = 0$ to $\Delta^* = 190$ ns for $i_{\mathrm{p},0} = 85^\circ$ after a year. This suggests that the frequency offset (Δf^*) is at the level of $\lesssim 6\times10^{-15}$ (see Table 2 for details). This de-synchronization (Δ^*) and frequency offset (Δf^*) demonstrate the deviations of the time and frequency of a clock in a time-aligned orbit, whose model takes the Moon's point-mass and J_2^M terms only, from a more realistic one that includes more gravitational perturbations from the Sun, the planets, and high-order harmonics of the Moon. We compared this with the uncertainty when realizing the selenoid time in O2, finding that the offset (Δf^*) of the time-aligned orbit is just 3.75% of the frequency difference of 1.6×10^{-13} in O2 due to the high variations of the lunar surface topography (Bourgoin et al. 2026). This suggests that realizing O2 by deploying clocks in the time-aligned orbit would be less susceptible to interference from natural causes than by landing clocks on the lunar surface.

We hypothesize that the deviation of the mean orbital elements in our numerical simulations from those required by the time-aligned orbit (Eq. (8)) causes the de-synchronization and frequency offset depicted in Fig. 1a. To test this hypothesis, we corrected the simulated τ^*_p, Δ^*, and Δf^* as

$$\tau^*_\mathrm{p,c} = \tau^*_\mathrm{p} - \Delta L_\mathrm{P}\,\tau^*_\mathrm{p}, \tag{14}$$

$$\Delta^*_\mathrm{c} = \tau^*_\mathrm{p,c} - \tau_\mathrm{s}, \tag{15}$$

$$\Delta f^*_\mathrm{c} = \Delta f^* - \Delta L_\mathrm{P}, \tag{16}$$

with

$$\begin{aligned}\Delta L_\mathrm{P} &= L_\mathrm{P}(\bar{a}^*_\mathrm{p}, \bar{i}^*_\mathrm{p}) - L_\mathrm{P}(\bar{a}_\mathrm{p}, \bar{i}_\mathrm{p}) \\ &= -\frac{3}{2}\frac{GM_\mathrm{M}}{c^2\bar{a}_\mathrm{p}}\frac{\Delta a}{\bar{a}_\mathrm{p}} \\ &\quad -\frac{21}{2}J_2^\mathrm{M}\frac{GM_\mathrm{M}}{c^2\bar{a}_\mathrm{p}}\frac{R_\mathrm{M}^2}{\bar{a}_\mathrm{p}^2}\left(1-\frac{3}{2}\sin^2\bar{i}_\mathrm{p}\right)\frac{\Delta a}{\bar{a}_\mathrm{p}} \\ &\quad -\frac{21}{2}J_2^\mathrm{M}\frac{GM_\mathrm{M}}{c^2\bar{a}_\mathrm{p}}\frac{R_\mathrm{M}^2}{\bar{a}_\mathrm{p}^2}\sin\bar{i}_\mathrm{p}\cos\bar{i}_\mathrm{p}\,\Delta i \\ &\quad +O[(\Delta a)^2,\ \Delta a\,\Delta i,\ (\Delta i)^2],\end{aligned} \tag{17}$$

where L_P is defined in Eq. (6), $\bar{a}^*_\mathrm{p}$ and $\bar{i}^*_\mathrm{p}$ are the mean elements obtained by averaging outcomes of numerical simulations, and we neglect the non-linear effects of $\Delta a = \bar{a}^*_\mathrm{p} - \bar{a}_\mathrm{p}$ and $\Delta i = \bar{i}^*_\mathrm{p} - \bar{i}_\mathrm{p}$. Since $J_2^\mathrm{M} \ll 1$, we believe the first term of Δa in Eq. (17) plays the most important role there. Figure 1b shows that the absolute values of corrected Δ^*_c are no more than 13 ns after a year and that the absolute corrected Δf^*_c is no more than 4×10^{-16}. This suggests that a more careful deployment of a clock into the time-aligned orbit could improve O2 performance by a factor of 10.

4. Conclusions and discussion

In the context of defining the LRT and the challenges for landing clocks on the surface of the Moon, we show that there exist time-aligned orbits around the Moon with semi-major axes of about 1.5 lunar radii. The readings of an ideal clock in such an orbit can equal the selenoid time, and the same readings could easily be converted to the TCL via a known linear transformation. Therefore, it could be possible to simultaneously realize the LRT options O1 and O2 of Bourgoin et al. (2026) with a single orbital clock. To assess its performance, we conducted a set of numerical simulations. We find that the proper time in the time-aligned orbit under a more realistically lunar gravitational environment would de-synchronize from the selenoid time by up to 190 ns after a year with a frequency offset of 6×10^{-15}, which is only 3.75% of the frequency difference in O2 caused by the lunar surface topography. Meanwhile, if we can account for the deviation of the mean orbital elements in our simulations from those required by the time-aligned orbits, we would reduce the de-synchronization and frequency offset by an order of magnitude to 13 ns and 4×10^{-16}.

The terrestrial planets may also have their own time-aligned orbits (see Table 3). This would mean that it might be possible to realize the reference times of planets beyond the Earth–Moon system with clocks in these orbits. This shows that options based on the time-aligned orbits are scalable, meaning we could avoid the risks associated with landing clocks on the surfaces of these planets.

Table 2. Comparison of nominal mean orbital elements ($\bar{\sigma}_{\rm p}$) and mean orbital elements ($\bar{\sigma}^*_{\rm p}$) from our numerical simulations with $\sigma = \{a,\ e,\ i\}$.

Number	$\bar{a}_{\rm p}$ [km]	$\bar{e}_{\rm p}$	$\bar{i}_{\rm p}$	$\bar{a}^*_{\rm p}$ [km]	$\bar{e}^*_{\rm p}$	$\bar{i}^*_{\rm p}$	Δ^* [ns]	Δf^*	$\Delta^*_{\rm c}$ [ns]	$\Delta f^*_{\rm c}$
1	2606.2658	0	0	2606.1094	0.0039	1.907°	50	1.6×10^{-15}	−6	-2×10^{-16}
2	2606.1186	0	25°	2605.9063	0.0056	23.395°	70	2.3×10^{-15}	−13	-4×10^{-16}
3	2605.7163	0	54.736°	2605.3468	0.0046	53.475°	140	4.4×10^{-15}	−9	-3×10^{-16}
4	2605.4477	0	85°	2604.9803	0.0040	84.715°	190	6.0×10^{-15}	9	3×10^{-16}

Notes. In our simulations, we adopted each initial state as $\sigma_{\rm p,0} = \bar{\sigma}_{\rm p}$. The resulting and corrected frequency offsets and de-synchronizations are also listed. The de-synchronizations Δ^* and $\Delta^*_{\rm c}$ are shown as their values at the end of a year.

Table 3. Mean semi-major axis ($\bar{a}_{\rm p}$) of the time-aligned orbits for four terrestrial planets.

Planet	$\bar{a}_{\rm p}$ [km]
Mercury	3660.097
Venus	9097.728
Earth	9556.250
Mars	5087.696

Notes. In the above cases, the orbit inclination ($\bar{i}_{\rm p}$) is set to 0.

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